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On Becoming a Graduate Student Teacher of Mathematics

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On Becoming a Graduate Student Teacher of Mathematics

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Abstract

Project Summary

As a nation, we face a critical need to expand and enhance the STEM workforce and to improve the quantitative literacy of our populace. Doing so requires effective instruction in mathematics at all levels, from pre-kindergarten through post-graduate studies. However, while much work has been done to improve K-12 teacher preparation in mathematics, and to help new faculty members learn to balance research and teaching duties, we are only beginning to address the preparation of mathematics graduate students to teach. Moreover, those limited efforts have, until recently, been largely focused on imparting teaching strategies rather than on developing a teacher identity. Transformative change in STEM education requires that university faculty members integrate teaching as a key component in professional identity. Identity is most fluid during transition stages, and graduate education is certainly that. By understanding how mathematics graduate students situate teaching within their developing professional identity as a mathematician, we may begin to understand how to bring about lasting change.

This project describes a mixed-methods multiple case study. Four first-year mathematics graduate students participated in a semester-long teaching seminar jointly attended by 18 pre-service secondary mathematics education majors. The graduate students and four of the undergraduates assisted with classroom instruction in precalculus courses at the university. Seminar meetings were designed to foster communication between the groups and to support develop of a teaching identity in both groups. The study seeks to understand how those four graduate students experienced teaching in their first year of graduate school.

Intellectual Merit

Currently, no framework exists for understanding the development of a teacher identity among mathematics graduate students. The results of this study may help in the adaptation of existing frameworks for teacher identity among preservice secondary mathematics teachers; and for professional identity among secondary teachers, clergy and psychologists. Such a framework could then serve as one component for evaluating professional preparation of mathematics graduate students. In addition, it could assist in the development of effective materials for inducting mathematics graduate students into a professional community that embraces teaching as one aspect of scholarship.

Broader Impacts

Graduate students bear a heavy share of the introductory mathematics teaching load at institutions around the country. Improving their preparation to teach will have a direct impact on the quality of STEM education for the general populace. Since those graduate students go on to form the mathematics faculty across the nation, encouraging them to develop a professional identity that includes teaching as a core component will arguably have a lasting impact on STEM education nationwide. An imperative first step in creating such transformative change is understanding how graduate students perceive teaching.

Dedication

This dissertation is dedicated to my children, Molly and Aidan.

Acknowledgments

My committee members were supportive and patient over several years. Their questions and challenges enriched this dissertation immeasurably. Their friendship and support buoyed me through personal challenges. I look forward to many years of professional collaboration with each of them.

My parents, Ann and Jack Parkhurst, provided emotional, financial, and temporal support particularly during the final year of this project. My children, Molly and Aidan, were patient and caring beyond their years. To these, my closest family, I owe more than I can say.

Jennifer Van Dyken undertook this journey with me, from the day we both enrolled in *Research Methods* to the day I presented the dissertation for defense. Without her moral support, occasional harassment, and steadfast friendship, I might have walked away at any one of several times the task seemed insurmountable. *Thank you, Jenn.*

Sue Watts, Neil Calkin, and Toran and Sarah Gordinier were unfailing in their friendship, even when I was at my grumpiest and least lovable. I am grateful not only for their support at key moments, but also for the fact that they are still willing to listen to me babble.

Three consecutive department chairs approved my undertaking this work while employed full-time in Mathematical Sciences. Without their approval, and supportive scheduling from two coordinators of instruction, I would have been unable to complete this work. Thank you to Robert Taylor, Jim Coykendall, Christopher Cox, Calvin Williams, and Judith McKnew.

I would also like to gratefully acknowledge the four graduate students who consented to be the focus of this dissertation. Without their involvement, time, and candid reflection, this work could not have been undertaken.

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Chapter 1

Introduction

1.1 Background Information

In 1990, Ernest Boyer authored a call to reconsider the priorities of the academic profession [Boyer, 1990]. He called for balance and integration between four types of scholarship: discovery, integration, application, and teaching. Twenty-five years later, institutions have largely embraced Boyer's first three types of scholarship under the broader umbrella of "research". However, while teaching is generally included as a tier one criterion for faculty review, many doctoral-granting institutions follow an unwritten practice of weighting research productivity more heavily in tenure, promotion and retention decisions. Indeed, such institutions vie for classification as a "very high" research institution under the Carnegie classification of institutions. Those classifications are based on research and development expenditures, research and development staff, and doctoral degrees granted. The definition of "research and development" used in the data collection for STEM departments specifically excludes funding for teaching-related scholarship [Carnegie Institution, 2016]. In 2014, Krause published results of an interview-based study whose goal was to "establish an empirical basis for understanding how academic disciplinary cultures affect the nature and quality of teaching and learning in higher education" [Krause, 2014]. Of the 11 academic mathematicians interviewed, only four perceived themselves as "part of a teaching community within their discipline."

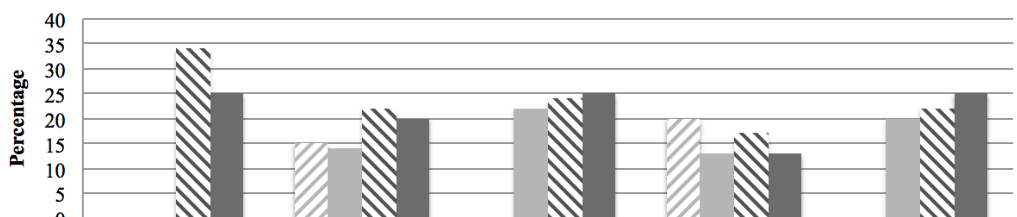
Some universities have created separate tenure-track paths for "teaching professors" with tenure and promotion based primarily on the scholarship of teaching. Such tracks, rather than encouraging the integration of types of scholarship, further separate them, implicitly stating that one cannot carry out both a successful research program and a successful scholarship of teaching. As Boyer noted in 1990, though,

> [G]ood teaching means that faculty, as scholars, are also learners ... teaching, at its best, means not only transmitting knowledge but transforming and extending it as well ... In the end, inspired teaching keeps the flame of scholarship alive ... What we urgently need today is a more inclusive view of what it means to be a scholar – a recognition that knowledge is acquired through research, through synthesis, through practice, and through teaching.

Four years after Boyer's report, the American Mathematical Society introduced Project NExT (New Experiences in Teaching) in recognition of the need to help new faculty members integrate teaching into their professional identities [Gallian et al., 2000]. Since 1994, nearly 1500 new Ph.D. mathematicians have participated in the program. Entry into the first faculty position is certainly a critical transition point in the professional workforce timeline, and influences at this stage can impact the professional identity. However, the bulk of professional identity formation for graduate students occurs during the graduate program of study itself [Hirt and Muffo, 1998, Holmes, 2015, Kajfez and McNair, 2014]. Interventions during those years are arguably more impactful than those at the transition into a faculty role. Indeed, early teaching experiences have been shown to play a major role in enduring teaching practice [Boice, 1996, VanZoest et al., 2012].

There have certainly been many laudable programs aimed at improving the professional preparation of graduate students and new faculty members, particularly among the STEM disciplines [Austin et al., 2008, Gallian et al., 2000, Golde, 2008, Harper et al., 2013]. The Preparing Future Faculty Initiative by the Council of Graduate Schools funded multiple programs for exactly this end. One outcome of those pilot programs was a specific call for changes in the preparation of future mathematics and science faculty [Pruitt-Logan et al., 2002]. More recently, that same initiative took on the narrower focus of preparing future faculty to assess student learning across multiple disciplines [Denecke et al., 2011]. Among the changes urged by these two reports is a call to change the academic culture in such a way that the false "teaching vs. research/scholarship" dichotomy is shifted to one of "teaching as research/scholarship" [Denecke et al., 2011]. As part of that, institutions are charged with helping graduate student develop more effective teaching strategies and a better understanding of students as learners [Pruitt-Logan et al., 2002].

These calls for a paradigm shift are grounded not merely in theoretical ideals but also in pragmatism. A significant percentage of undergraduates have graduate students as instructors for their mathematics courses, among both STEM and non-STEM majors (see Figure 1.1.1 [Lutzer et al., 2002, Lutzer et al., 2007, Blair et al., 2013]. Since performance in STEM-related courses is a key factor in attrition rates for STEM majors [Chen and Soldner, 2013], the preparation of mathematics graduate students to teach becomes a critical issue in examining pathways to success, particularly for undergraduate STEM majors. Moreover, most post-secondary mathematics faculty members, regardless of the type of institution at which they are ultimately employed, have their first mathematics teaching experience at a doctoral-granting research institution. Thus, understanding that first experience for graduate students has much broader implications than simply improving practice at doctoral-granting institutions themselves.



	Introductory courses (non-calculus)	Mainstream Calculus 1	Non-mainstream Calculus I	Mainstream Calculus II	Non-mainstream Calculus II
1995		15		20	
2000		14	22	13	20
2005	34	22	24	17	22
2010	25	20	25	13	25

Figure 1.1.1: Graduate Students Teaching Undergraduates. Percentage of undergraduates taught by graduate students, broken down by course and year. Data drawn from the reports from the Conference Board of the Mathematical Sciences citecbms2000, cbms2005, cbms2010.

Nonetheless, the professional development of mathematics graduate teaching assistants remains under-examined from a research perspective. Speer, Gutmann, and Murphy's 2005 review of the published research on professional development of mathematics graduate teaching assistants (GTA's) concluded that there was essentially no such research [Speer et al., 2005]. It is also telling that although there is a considerable body of research on development of teacher identity among preservice secondary mathematics teachers, *c.f.* [VanZoest and Bohl, 2005, Lutovac and Kaasila, 2014, Hodges and Cady, 2012, Cavanagh and Prescott, 2007, Gottlieb, 2012, Beauchamp and Thomas, 2011, Ward et al., 2011, Beijaard et al., 2004, Beauchamp and Thomas, 2009, Flores and Day, 2006, Lasky, 2005, Watt et al., 2007, Gellert et al., 2013], there are to date very few peer-reviewed research papers addressing development of teacher identity among mathematics graduate students [Beisiegel and Simmt, 2012] and/or teaching orientations among STEM graduate students [Gilmore et al., 2014]. The latter paper was framed within research on K-12 teachers and on general graduate student development because of the paucity of research on mathematics graduate student teacher identity development.

Professional programs for secondary mathematics teachers have long held the goals now proposed for graduate student preparation. Indeed, secondary education certification in most states requires significant coursework in learning theory and pedagogy and a student teaching experience or mentored induction process ranging from one semester to three full years [Goldrick, 2016, Dossey et al., 2012]. It is not practical to require GTAs to complete 30 or more units of education coursework along with their subject matter coursework and mathematical research requirements, nor is it appropriate: the student population for secondary mathematics and for university teaching are quite different, as are the contextual issues associated with higher education. However, it is worth examining effective practice in the preparation of secondary teachers to see what components might be translated appropriately to graduate student preparation.

In order to develop effective experiences that encourage the inclusion of the scholarship of teaching as an integral view of what it means to be an academic mathematician, we must first understand how mathematics graduate students navigate the role of teaching in their graduate program. Graduate school is a time of enormous change and enormous pressure. During their time in graduate school, students are expected to make the transition from students in their discipline to practicing researchers in their discipline. Doing so requires that they not only master a body of skills and knowledge, but also that they internalize what it means to be a professional in the discipline and how to balance the roles of research and teaching. Thus, the question of identity – the interplay between how one perceives oneself and how one interacts with others in various settings – must be at the heart of virtually any examination of the graduate school experience [Austin, 2002, Hirt and Muffo, 1998, Byers et al., 2014, Janke and Colbeck, 2008, Austin et al., 2008, Gardner, 2008, Murray, 2000, Golde, 2008]. Within the very large field of identity theory, established frameworks for communities of practice, teacher identity, and professional identity development are particularly relevant to the research questions at the heart of this study.

1.2 Research Questions

This study was developed to address specific aspects of the broader question

> *How do mathematics graduate students experience the phenomenon of their first teaching experience in graduate school?*

. We established an enriched teaching community by embedding four first-year graduate students in a teaching seminar with eighteen preservice secondary mathematics teachers and incorporating best practices from K–12 mathematics teacher preparation. Within that context, we specifically considered the following questions:

1. What messages do first-year graduate students at Clemson receive, and from whom, about the role of teaching in the professional identity of mathematicians?
2. What sources of information do graduate students rely on as they navigate expectations during the first year in graduate school?
3. What aspects of teacher identity are reinforced or weakened by first-year graduate school experiences, and specifically by the enriched teaching experience?
4. What impact do future goals have on how graduate students balance first-year expectations in graduate school?
5. What impact does the enriched first-year experience have on subsequent teaching practice?

1.3 Communities of Practice

A group of people interacting on the basis of a common goal or interest forms a *community of practice* [Lave and Wenger, 1991]. In simplifed form, newcomers to the community start as *peripheral* members. Over time, as they observe and subsequently model the accepted behaviors of the community, they move to a more central position as a *regular* member. Some regular members go on to become *core* members, central to the life and practice of the community (see Figure 1.3.1). The path an individual takes within a community of practice, including whether a peripheral member moves into a more central role, remains peripheral, or departs the community entirely, is dependent on many factors beyond the scope of this dissertation.

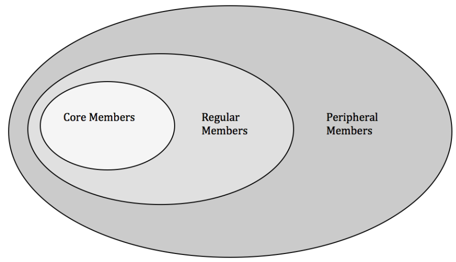


Figure 1.3.1: Lave and Wenger's Model for Communities of Practice

Indeed, community of practice models specific to particular populations often establish far more nuanced levels within the simplified shells modeled here, and may include complex interactions among and between those levels, c.f. [Orbe, 2004, Heyd-Metzuyanim and Sfard, 2012, Owens, 2008, Peressini et al., 2004, Cavanagh and Prescott, 2007, Varelas et al., 2005, Tonso, 2006, Sexton, 2008, Kajfez and McNair, 2014] for just a few of the studies specific to science students, undergraduate engineering majors, preservice secondary teachers, graduate students, and new teachers. No detailed model currently exists for the mathematics graduate school community of practice, nor do we seek to establish one in this dissertation. Rather, we seek to pull the most relevant pieces from the existing research on communities of practice in order to establish multiple frameworks for examining the data in addressing our research questions.

Lave and Wenger's work was rooted in a view of learning as situated within a community of practice [Lave and Wenger, 1991], and this view of *situated learning* or *situated practice* informs many of the current models for identity within a professional domain. The situated learning for secondary teachers occurs within a community whose core members have a primary professional identity of 'teacher.' That is not the case for mathematics graduate students. They have chosen to become peripheral members is a community whose core members have a primary professional identity as 'mathematician' and thus we presuppose that the graduate students are disposed to imitate the attitudes and behaviors of *mathematicians* [Hirt and Muffo, 1998, Janke and Colbeck, 2008]. In 1999, Burton conducted an extensive interview-based study of university mathematicians and situated the results within a framework of community of practice [Burton, 1999]. The picture that emerged was one of a community in which contributing to the frontiers of knowledge remains of paramount importance and in which there is a strong disconnect between practice of mathematics and teaching of mathematics. He notes, “[a]lthough the interviews were not about teaching and learning, often, after they were over, we had conversations in which the participants made clear how little they thought about their teaching.” Moreover, an extensive review of the literature on college mathematics teaching reveals that very little is known about the teaching practices of postsecondary mathematics faculty [Speer et al., 2010].

At the same time as they adjust to more advanced coursework and research expectations, mathematics graduate students are taking on teaching duties and developing situated practice as teachers of mathematics within the mathematical community of practice [Harris et al., 2009, Park, 2004, Staton and Darling, 1989]. Often their first-year teaching supervisors are no longer active in mathematical research. Where these individuals fit within the mathematical community of practice is at the time same intriguing, unclear, and beyond the scope of this dissertation. We attempt to shed some light on this through two of our research questions (from whom graduate students receive messages about the role of teaching, and what sources of information first-year graduate students rely on) but we are far from constructing a full model of the community or the role of lecturers within that community, nor is that the focus of that study. While keeping in mind what little has been established about core members of mathematical communities of practice and acknowledging the lack of knowledge about who constitutes regular members of that community, much less what the behavior and attitude of those regular members might be with respect to teaching, we seek to select the most appropriate established models for teacher identity among mathematics graduate students.

We wish to explore the specific aspect of teaching identity situated within the larger context of professional identity as a mathematician. There is no body of research addressing this specific population. Instead, since mathematics graduate students are just entering the profession of teaching, and since their first teaching assignments are often teaching mathematics content that overlaps with high school mathematics curricula, we draw in part from the extensive literature on secondary teacher identity development [VanZoest and Bohl, 2005, Ball et al., 2008, Beijaard et al., 2000, Beijaard et al., 2004, Hamman et al., 2010]. However, we recognize that these frameworks do not transfer directly since they are situated among populations for whom the central figure within the community of practice is that of "teacher". We therefore pull also from the research literature on professional identity development from a situated practice perspective [Ronfeldt and Grossman, 2008, Ibarra, 1999, Markus and Nurius, 1986]. In distilling this research down to the most useful pieces, we find that two different classes of model – static models of teacher identity and dynamic models of identity development – each play a critical role in understanding the experiences of mathematics graduate students as they progress through their first-year experiences.

1.4 Static Models of Teacher Identity

Beijaard, Verloop, and Vermunt decomposed teacher identity into three categories based on a personal knowledge perspective: teacher as subject-matter expert, teacher as pedagogical expert, and teacher as didactical expert [Beijaard et al., 2000]. Subject-matter expertise pertains to understanding of the content to be taught. Didactical expertise refers to constructing and delivering a learning experience, and to assessing outcomes of the instructional practice. Pedagogical expertise refers to supporting the emotional and social needs of students, and to adapting instruction to fit individual learners needs. Individuals are located within this framework based on relative importance they assign to each of these types of expertise. Throughout this dissertation, we will for simplicity refer to this model as the *Beijaard Triangle* or as *Beijaard's triangular model*.

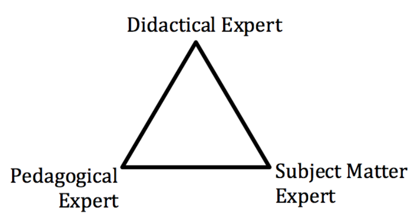


Figure 1.4.1: Beijaard, Verloop, and Vermunt's Model of Teacher Identity, developed for experienced secondary teachers irrespective of field. "S" indicates subject-matter expertise, "D" indicates didactical expertise, and "P" indicates pedagogical expertise.

Perservice and new secondary mathematics teachers typically cluster more closely along the pedagogical-didactical axis, with experienced secondary teachers balancing the three types of expertise and thus moving towards the center of the triangle. Mathematics graduate programs traditionally take the implicit view that a subject-matter expert with minimal on-the-job training will be an effective teacher. To the extent that additional training is provided for mathematics GTA's, that training falls largely within the "didactical expert" branch of this model [Speer et al., 2005]. Thus, most research mathematicians and mathematics GTA's likely fall along the subject-matter/didactical edge of Beijaard's triangle. This model provides insight into the static underlying structure within which the community of research mathematicians and the community of practicing secondary teachers largely operate. Our adaptation of this static model for use as a dynamic model of identity development for this study is novel and is described in Section 4.1.

There has been considerable research into the link between content knowledge and effective teaching, c.f. [Adler and Davis, 2006, Ball and Bass, 2004, Ball et al., 2008, Copur-Gencturk and Lubienski, 2013, Doig and Groves, 2007, Gningue et al., 2013, Shulman, 1986]. Given the importance of the role of subject-matter expert within the view of teaching among research mathematicians, we had hoped to use Ball, Thames, and Phelps' model of content knowledge for teaching [Ball et al., 2008] as the primary framework for analyzing teacher identity among the mathematics graduate students. This model, developed for use with experienced K-12 mathematics teachers, divides content knowledge into six categories in two groupings (See Figure 1.4.2), which we will refer to as *Ball's pie*.

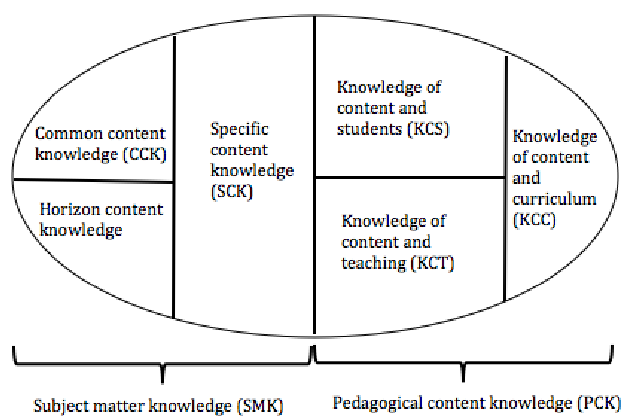


Figure 1.4.2: Ball, Thames, and Phelps's Model of Content Knowledge for Teaching, developed for experienced K-12 mathematics teachers.

- Subject matter knowledge:
  - CCK: knowing how to carry out the mathematics in settings aside from teaching,
  - HCK: knowing how mathematical topics are related to one another, and
  - SCK: knowing how to "unpack" the mathematical content and skills in such a way that you can teach the mathematical content of a course.

- Pedagogical content knowledge:
  - KCS: knowing how students think about the mathematical content of the course,
  - KCT: knowing how to design instruction to teach the content of the course, and
  - KCC: knowing how the content of one course fits within a larger curricular scope.

Mathematics graduate students arguably enter their first teaching experience with a reasonable CCK, and strongly rooted at the "subject-matter expert" pole of Beijaard's triangle. We anticipated that as their teaching identity formed within the context of the graduate program, we would see development into the KCS and KCT regions of Ball's pie. That the progression would have supported integration of the two static models and connection to one of the dynamic teacher identity development models described below. However, the experience base of the graduate students was insufficient to allow them to reach independently the pedagogical content knowledge side of the model during the first year, and this model proved entirely unsatisfactory for this population. We anticipate that future work with third- and fourth-year mathematics graduate students will justify integration of this critical model of mathematics teacher identity.

1.5 Dynamic Models for Identity Development

1.5.1 Van Zoest & Bohl

Van Zoest and Bohl provide a framework for the formation of mathematics teacher identity (see Figure 1.5.1) within the trajectory of communities through which secondary mathematics teachers pass as they progress from student to practitioner [VanZoest and Bohl, 2005]. Their model considers the interplay between individual knowledge and beliefs, and situated practice within a teaching and learning community. They distinguish between *knowledge* and *beliefs/commitments/intentions* by considering knowledge as that which is generally accepted as true and not open to debate, while beliefs/commitments/intentions are open to debate and thus must be negotiated within the community of practice. That negotiation occurs through engaging in shared practice, discussing roles and actions, and evaluating the outcomes of the shared practice. The spectrum from 'self-in-mind' to 'self-in-community' is subdivided into content, pedagogy, and professional participation domains.

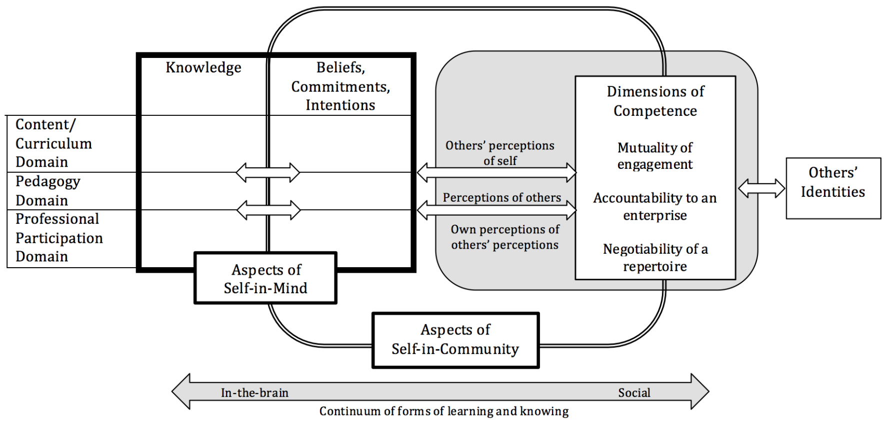


Figure 1.5.1: Van Zoest and Bohl's Model of Teacher Identity Development, developed for secondary mathematics teachers from student teaching through the first few years of teaching practice.

The content domain in Van Zoest & Bohl's model aligns well with the subject-matter expertise pole of Beijaard's triangle, and with the CCK and HCK levels of Ball's pie. All three models define that dimension as pertaining to the content matter of the course. Van Zoest & Bohl's pedagogy domain, however, clusters together Shulman's competencies of general pedagogical knowledge, pedagogical content knowledge, and knowledge of learners [Shulman, 1986]. In short, this domain focuses on who should be taught and how to teach them and it therefore roughly includes both the pedagogical expertise and the didactical expertise poles of Beijaard's triangle, and includes the SCK, KCS, and KCT wedges of Ball's pie. The third domain of Van Zoest & Bohl's model, professional participation, is tied to Shulman's knowledge of educational contexts and knowledge of educational ends [Shulman, 1986], which together form the KCC piece of Ball's pie, and which are absent from Beijaard's triangle. This domain is essentially knowing how one's practice fits within the larger professional teaching and university communities.

However, the teaching community trajectory for mathematics GTAs is lacking nearly all of the elements present to support secondary teachers in their transition through the teaching community of practice. As a first-year graduate student, a GTA may receive some mentoring and support from a supervising faculty member in the class to which the GTA is assigned. By the second year, most mathematics graduate students serve as instructors of record for their own classes and are assumed to be functioning independently. Most graduate programs in mathematics offer little to no ongoing structured mentoring for graduate students beyond the first year and virtually no mathematics graduate programs provide or require courses to develop a knowledge base for teaching in general [Deshler et al., 2015]. Without an established series of teaching communities of practice within which students interact, the "aspects of self in community" portion of Van Zoest and Bohl's framework for mathematics teacher identity contributes less to the feedback loop of teacher identity development in general. Nonetheless, certain aspects of this model are useful for analyzing mathematics teacher identity development among graduate students in general and our research design in particular includes components specifically intended to create at least a temporary teaching community of practice for the study participants.

1.5.2 Ronfeldt & Grossman

Although the "perpetual graduate student" certainly exists, the vast majority of students begin graduate studies with the intention of completing those studies and moving on to something else. They are motivated not by the graduate school experience itself, but rather by what lies at the end of graduate school. Their current choices are largely driven by goals for the future [Simons et al., 2004, Husman and Lens, 1999, Lens et al., 2012] and by who they see themselves becoming [Cross and Markus, 1991, Markus and Nurius, 1986].

As graduate students move from a peripheral role to a more central role to an eventual identity as "mathematician", they must negotiate decisions on time management and use of their limited resources. The requirements of their own coursework to gain mathematical knowledge and skills must be balanced with the requirements of teaching to retain an assistantship, and with research requirements. In addition, many have family or other social responsibilities that draw on the limited time and money resources [Byers et al., 2014]. As they traverse the span from periphery to central role, the path they take and the choices they make are influenced by how they view their current location within the community of practice, as well as by how they view the central figure towards which they are moving. That is, they may go through a series of "provisional selves" [Ibarra, 1999, Hamman et al., 2010] as they solidify a professional identity.

Ronfeldt and Grossman offer a framework (see Figure 1.5.2) for identity development in professional education programs for teachers, clinical psychologists, and clergy that provides a useful lens through which to analyze aspects of teacher identity formation among mathematics graduate students [Ronfeldt and Grossman, 2008]. We will also use “fieldwork contexts” in place of “teaching contexts”, and “research contexts” in place of “supervisory contexts” in adapting this framework for our analysis.

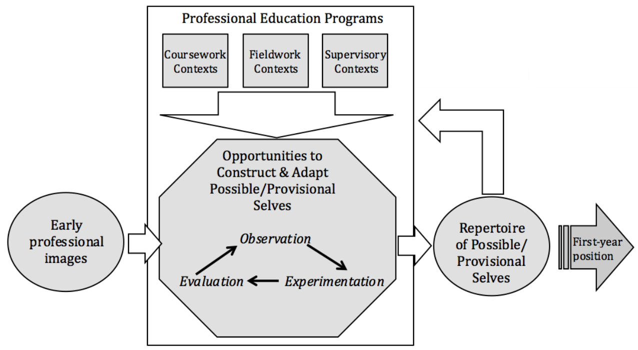


Figure 1.5.2: Ronfeldt and Grossman’s Model of Professional Identity Development, based on participants in professional education programs in clinical psychology, clergy, and secondary education.

1.6 Best Practices from Secondary Teacher Preparation

Given the extensive literature on secondary mathematics teacher identity development and effective practice, it is reasonable to draw from that community to identify best practices that might form an effective scaffold for supporting development of teacher identity among mathematics graduate students. We do not have the luxury of requiring graduate students to take entire courses on pedagogy for postsecondary mathematics instruction; the practices we select must fit into limited time and deliver the maximum impact. For this study, we identified student teaching, case study, and lesson study as three practices that have demonstrated impact on teacher identity and practice among secondary teachers. The modifications made to each of these for implementation are described in Chapter 2.

1.6.1 Situated Practice (Student Teaching)

A long-established practice in K-12 teacher preparation is student teaching, in which the novice teacher is placed in a cooperating teacher’s classroom to begin teaching in a supported environment, gradually taking on more duties for planning, implementation, and evaluation of lessons and student mastery. Considerable research points to the importance of the student teaching experience in developing both effective instructional practice and a strong teacher identity [Jarvis-Selinger et al., 2010, Ward et al., 2011, Beauchamp and Thomas, 2011, Horn et al., 2008].

In mathematics, graduate teaching assistants typically spend the first year in a support role that includes a combination of grading, office hours, and conducting lab study sessions. Their role in planning or delivering instruction is typically minimal [Harris et al., 2009, Park, 2004] as is the feedback they receive on their limited teaching practice [Shannon et al., 1998, Boyle and Boice, 1998]. Most accrediting bodies require completion of 18 hours of graduate coursework in order to teach in a discipline. Thus on entering the second year of graduate studies, teaching assistants are now "certified" to teach and are typically assigned one or more sections of their own course, going from minimal involvement in the practice of teaching to full responsibility for a course with no intermediate steps. Since graduate students are not accredited to teach during their first year of graduate studies, it would be within the scope of their normal duties for a teaching assistant's first year to involve experiences that more closely resemble that of student teaching. Our research design implements a limited situated practice model of student teaching in a hybrid classroom structure during the first semester.

1.6.2 Case Study

The use of cases in professional preparation has a long history, not only in law, business, medicine, and engineering, but more recently in K – 12 teacher preparation [Colbert et al., 1996, Burgoyne and Mumford, 2001, Christensen and Hansen, 1987, Garvin, 1993, Sykes, 1989, Wassermann, 1994]. It has also been suggested as an effective method for preparation of graduate students [Allvine et al., 2007]. The essence of case study is to allow pre-professionals to wrestle with complex issues from practice prior to entering that professional practice themselves. In that way, they develop a more nuanced understanding of the field and, to put it in terms of communities of practice, they gain insight into the behavioral norms of core members of the community, thus accelerating their movement from peripheral to central.

Cases appropriate for use with mathematics graduate students must focus on the mathematics they will be teaching, which limits the options as there are very few such cases in publication. The

Harvard Mathematics Case Development Project (HMCDP) sought in the 1990's to establish a base of cases for the preparation of secondary mathematics teachers. Several of those cases were shortened and published as *Windows on Teaching Math: Case Studies in Middle and Secondary Classrooms* [Merseth, 2003]. In 2001, Friedberg et al. published a set of fourteen cases intended for use specifically with mathematics graduate students [Friedberg et al., 2001]. There is little published research on their use in practice, although informal conversations at conferences indicate that they are being used at some institutions. Despite extensive review of the literature, these cases did not come to light until after the implementation of the research design. Several of the cases remain viable despite vast changes in technology since 2001, and they are promising for future work.

To avoid confusion with the individual case analyses that form the basis of this multiple case study research project, we will use the term "cases" or "case arcs" when discussing the cases studied by the participants as part of the designed experiences.

1.6.3 Lesson Study

Lesson study has its origins in 19th century Japan where there was a national push to modernize education and incorporate Western educational methods while holding true to historical Japanese culture [Sarkar Arani et al., 2010]. It first came to the attention of the education community outside Japan in 1999 on release of the TIMSS Video Study [Stigler et al., 1999], and has since been widely adapted and adopted for teacher preparation with varying degrees of success [Doig and Groves, 2011, Lewis et al., 2009, Groves and Doig, 2007, Groves et al., 2013, Watanabe, 2002].

Traditional Japanese lesson study involves the development of a single lesson, generally around an hour in length, to be delivered by each member of the development team in turn over period often of several years. The goal is to achieve a "polished pearl" that can then be picked up and used by teachers throughout the country. Development of a lesson includes establishing specific goals for learning outcomes and understandings, identifying prerequisite knowledge, reviewing relevant research literature, and reflecting on practice to explore not only how students conceptualize the ideas embodied in the lesson but also how that lesson connects to a broader curriculum. At the heart of lesson study is development of the types of pedagogical content knowledge that form Ball, Thames & Phelps' model of teacher identity [Ball et al., 2008] as described in Section 1.4.

Chapter 2

Research Design and Methods

2.1 Institutional Review Board Approval

The work undertaken in this study was approved under Clemson University Institutional Review Board (IRB) number IRB-2014-306. Recruitment materials, statements of informed consent, survey instruments, and interview protocols were approved through the IRB and administered in a manner consistent with ethical human subjects research. Lesson study and case study artifacts and reflective writings were collected as part of regular course activity under protocols approved in the IRB application.

2.2 Research Design

The central research question we pose lends itself to a phenomenological study design. Phenomenological studies are intended to explore *how* people interact with lived experiences: what they perceive, how they act, and how they understand and interpret the experience. [Gubrium and Holstein, 2000]. Interpretive phenomenology then analyzes the data collected through a lens that seeks to accurately reflect the voice of the participants as interpreted by the researcher. Since we seek to understand how graduate students experience teaching during their first year of graduate school, we would ideally use an interpretive phenomenological approach, seeking in part to expose assumptions that may otherwise be taken for granted [Starks and Trinidad, 2007]. To conduct a phenomenological analysis, we would need to collect data from observations in context, from extensive in-depth interviews with the participants, and from artifacts generated by their lived experience. Those data would then need to be coded and analyzed to look for clustered themes and commonalities that would enable us to distill the core essence of the experience [Starks and Trinidad, 2007, Gubrium and Holstein, 2000, Gay et al., 2006].

However, the design of the study unavoidably placed the researcher in an evaluative role as instructor and faculty supervisor for the study participants for a full semester. It was thus not feasible to conduct a series in-depth interviews with intermediate analysis that would be called for in a true phenomenological study; doing so would have presented an unacceptable psychological risk to the participants. Instead, we conducted a mixed-methods multiple case study with four participants and analyzed the qualitative data through an interpretive phenomenological lens. Although the data was collected at multiple time points there was, by design, no intermediate analysis of data to inform the next steps in the study or the next type of data to be collected.

Data	**Type**	**Count**	**When Collected**	**Framework(s)**
Survey	Quantitative	2	Start of Semester 1	Van Zoest & Bohl, Ronfeldt & Grossman
			End of Semester 1	
Case Artifacts	Qualitative	7	Throughout Semester 1	Beijaard et al.
Lesson Study Artifacts	Qualitative	2	Middle of Semester 1	Beijaard et al., Van Zoest & Bohl, Ronfeldt & Grossman
			End of Semester 1	
Reflective Writing	Qualitative	2	Middle of Semester 1	Beijaard et al., Van Zoest & Bohl, Ronfeldt & Grossman
			End of Semester 1	
Interview	Qualitative	2	Middle of Semester 1	Beijaard et al., Van Zoest & Bohl, Ronfeldt & Grossman
			End of Semester 2	
Student Performance	Quantitative	1	End of Semester 3	Measure of teacher effectiveness

Table 2.2.1: Types of Data and Timeline for Collection

To address the research questions posed under the constraints of the design, we had multiple data collection points for a rich assortment of both quantitative and qualitative data (see Table 2.2.1). Each individual data collection is addressed in much greater detail in Chapters 3 and 4, with discussion of methods of analysis and results from each data type included there. In this section, we merely summarize the type, time point, and framework(s) used for analysis.

2.3 Institutional Background

Clemson University is a doctoral-granting university classified at the time of this study as a Research (Very High) institution. The Department of Mathematical Sciences was at that time housed within the College of Engineering and Science. There were then 51 tenure-stream faculty, 49 lecturers, 110 graduate students, and 262 undergraduate majors within the department. Mathematics graduate students were, and still are, almost exclusively supported by teaching assistantships. During their first year, GTAs support lower-division courses taught by lecturers and tenure-track faculty. During their second year and beyond, GTAs have historically had sole responsibility for their own small-enrollment (19 or fewer) sections of lower-division courses. Students nearing the Ph.D. have sometimes been given responsibility for a section of an upper-division course. During the time of this study, institutional pressures resulted in some graduate students being assigned three sections of a course in one semester, or being assigned sections of 45 students rather than the historical 19 or fewer.

Lower-division courses with which GTAs may be involved are differentiated based on major. STEM majors take one or more of Precalculus, Calculus I - III, and Differential Equations. Calculus I is also offered as a two-semester Long Calculus A and B sequence. Non-STEM majors take one or more of College Algebra, Business Calculus I and II, and Essential Mathematics for the Informed Society. A typical historical teaching trajectory for a GTA has historically been:

- First year: assistant in Precalculus, College Algebra, Long Calculus, or Calculus I
- Second year: instructor of record for Business Calculus I and/or II
- Third year: instructor of record for Calculus I or Essential Mathematics
- Fourth year: instructor of record for Calculus II or Essential Mathematics

The first-year teaching assignments of the graduate students have not historically translated directly into mimicry of teaching methods and materials the following year. Rather, they were generally faced with a student population and course content quite different than that of their first-year experience. It was essential, then, that we identify key elements that were not tied to a particular course, but rather that reflect a broader view of teaching as a discipline. This study was designed with that historical trajectory in mind. Again, however, institutional pressures resulted in significant changes to the manner in which second-year graduate students were assigned to courses. Those changes impacted the nature and quality of the data on student performance in the second year to a limited extent.

2.4 Selection of Participants

Assignment to this study was based solely on availability during the meeting times for precalculus and a teaching seminar. Only five graduate students were available for both. Of those, four were full-time GTAs in their first year of graduate school. The fifth was a third-year graduate student on part-time assistantship who had previous experience as a teacher of record. The four first-year GTAs form the subjects of this study. One of those four participants had technically started graduate school at Clemson the previous spring, but he was not supported on assistantship and was taking courses that prior semester to remedy undergraduate deficiencies, so was not embedded in the culture of the graduate program until the first semester of this study.

Of the four subjects, two were male and two were female. All were native English speakers and U.S. citizens. Three had experience as mathematics tutors during their undergraduate students. One had obtained a teaching certificate but chosen to attend graduate school prior to pursuing high school teaching. None had ever been teacher of record for a course at any level. All four had attended private, four-year colleges for their undergraduates studies. One of the men and one of the women participated in a Summer Bridge Program intended to remedy academic deficiencies from undergraduate studies prior to beginning graduate work in mathematics. Three of the four entered the program intending to pursue graduate studies in statistics and the fourth in computational mathematics. The participants in the study had undergraduate GPAs of 3.52, 3.52, 3.95, and 4.0, as compared to a mean undergraduate GPA of 3.81 among all incoming mathematics graduate students that year.

Graduate students in the Department of Mathematical Sciences at Clemson University are initially admitted to the Master's program even if they intend to pursue a Ph.D. After completing six breadth courses, students may apply to transfer into the Ph.D. program and sit for preliminary examinations. Thus, all of the students in this study were nominally Master's students although three of the four entered the program intending to pursue a Ph.D.

2.5 Design of First Semester Experience

The first semester teaching experience for the graduate students in this project was designed to draw on best practices in K-12 teacher preparation. The goal was to give the graduate students extensive support to develop their pedagogical content knowledge and to foster a teaching identity within their broader professional identity, rather than to assess the effectiveness of a single intervention in isolation. We can then look to see how these students experienced teaching mathematics and to what extent they consider teaching as an integral part of their developing identities.

The four students who were subjects of this study were all assigned to assist with Precalculus. They were also assigned to take a teaching seminar that met concurrently with a senior-level course taken by 18 preservice secondary mathematics teachers (PSTs). The senior-level course, Exploration and Analysis of Secondary Mathematics (MATH 4080), is offered through the Department of Mathematical Sciences and is designed to focus on the mathematics of the secondary curriculum at a deep level. All of the PSTs took it concurrently with education capstone courses on teaching secondary mathematics, and immediately prior to the student teaching semester. One requirement of the concurrent education capstone course was placement in a cooperating teacher classroom at a local high school for 30 total hours over the course of the semester. Four of the PSTs also participated in a Creative Inquiry (CI) project for which they assisted once a week in the Precalculus classes to which the GTA's were assigned. MATH 4080 and the graduate teaching seminar met concurrently twice a week for 75 minutes each session.

Throughout the semester, students in the course and seminar worked in teams consisting of one GTA, one undergraduate enrolled in the CI course, and two or three undergraduates not participating in the CI (see Figure 2.5.1). There were five different team assignments over the course of the semester, ensuring that each GTA worked with each undergraduate CI participant and with all or nearly all of the non-CI undergraduates. The course and seminar revolved around lesson study and case analysis,

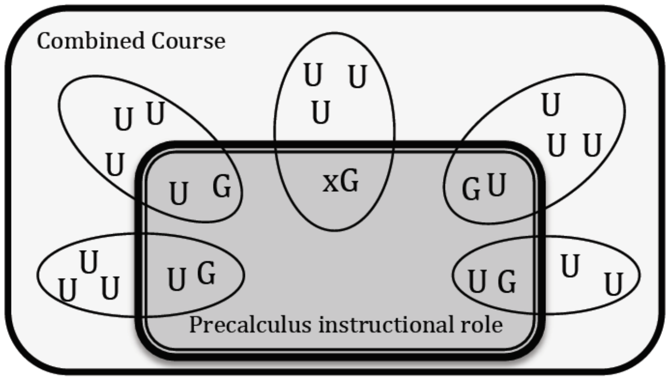


Figure 2.5.1: Spheres of Interaction for the Study Participants. Areas of interaction and influence for the undergraduate and graduate students in the study. The shaded area indicates the locus of influence of the researcher. "U" indicates an undergraduate student, "G" indicates a graduate teaching assistant and "x" preceding either designation indicates that the individual did not consent to the study.

each of which was connected to situated practice in Precalculus classrooms at the university and in cooperating teacher classrooms at local high schools. Pedagogical content knowledge was addressed directly and repeatedly in the course and seminar, as were reflection on practice and professional identity.

2.5.1 Case Arcs

NOTE: This section is drawn verbatim from a published paper [Gallagher et al., 2016].

In the 1990's, the Harvard Mathematics Case Development Project (HMCDP) sought to establish a basis of cases for the preparation of mathematics teaching professionals. Several of those cases were published as *Windows on Teaching Math: Case Studies in Middle and Secondary Classrooms* [Merseth, 2003], which we used as the sole required text for the combined course. Each of the published cases includes pre-case prompts, the case itself, and post-case prompts. Of the eleven cases available in the text, the six most closely linked to the content of the precalculus course were used for discussion in the combined course, as indicated in Table 2.5.1. One additional draft version of a case from the HMCDP was used as well; it is indicated in the table as unpublished.

Each of the seven case arcs consisted of a pre-case activity in class, individual reading of the case out of class, and in-class guided post-case discussion. The pre-case activities were primarily mathemati-

Case Title	Mathematical Content	Pedagogical & Contextual Issues
Lost in Translation	Order of operations, algebraic expressions, inequalities, verbal and symbolic representation systems	systematic conceptions and representations, questioning techniques, handling errors in student presentations
Slippery Cylinders	measurement, volume and surface area of cylinders, square roots, finite and infinite measures, estimation	counterintuitive thinking, student investigations, use of manipulatives, gender issues, group work
The Marble Line	discrete and continuous variables, modeling linear functions, interpreting slope and intercept in context, rate of change, absolute change	notion of variable, checking for understanding, heterogeneous grouping, questioning techniques, representational systems
Root of the Problem (unpublished)	radical expressions, factoring quadratics, solving radical equations, approximation, rational and irrational numbers	addressing student errors, assessing prerequisite knowledge, on-the-spot change of lesson plan, handling errors in student presentations
The More Things Change	exponential growth, evaluating exponential expressions, instantaneous and average rates of change	listening skills, small-class activities, discussion techniques, gender issues, competitive students
What is π Anyway?	area and circumference of circles and annuli, approximation, definitions of π and infinity, ratios without units	assessment, use of journals, grading procedures, effectiveness of models, representations for infinity
Ships in the Fog	parametric equations, minimization, systems of linear equations, scaling	confusion about units and scale, questioning techniques

Table 2.5.1: Case Analyses Used in the First Semester Experience. Case titles and content, in the order discussed in the combined course.

cal in that they were designed to have the combined course participants carry out the mathematical tasks of the case. In some instances, the pre-case activities also included pedagogical prompts. In general, the pre-case activities were conducted individually in class as an exit activity. The post-case discussion prompts fell into four broad categories: mathematical issues, analyzing student thinking, pedagogical issues, and contextual issues. Every case discussion included discussion of the mathematical issues and either student thinking or pedagogical issues (or both). Some post-case discussions included prompts from all four categories. For each post-case discussion, teams assigned a recorder for each prompt and recorded the contributions of each individual team member. Thus, we are able to look at the individual contributions made by each GTA in the discussion of each prompt. Although seven cases were used in the course, the mathematics for one case, Ships in the Fog, proved to be sufficiently challenging that the case discussion never reached pedagogical or contextual issues, but centered solely on developing the necessary mathematical skills.

In order to better understand the manner in which cases were used, and thus the nature of the data derived from case artifacts, we provide here fuller detail for a single case arc: Slippery Cylinders. This was the second case and it was covered in the third week of the semester when the bulk of the precalculus students were just starting to work on a course objective related to connections between geometry and algebra.

The pre-case activity was administered during the last twenty minutes of class. Groups were given two sheets of identical blank paper and instructed to create both a tall cylinder and a short cylinder by rolling the paper and taping together either the long edges or the short edges. Students responded individually, in writing, to these four prompts:

- If you were to pour puffed wheat into each of the cylinders your group just created, which cylinder would hold more puffed wheat?
- How could you determine the correct answer and convince someone else of its correctness?
- What understandings would someone else have to have in order to follow your argument or demonstration?
- What connections does this question about cylinders have to other areas of mathematics?

Students were permitted to leave as soon as they handed in written responses to all four prompts, and were expected to read the case carefully prior to the next class meeting. The post-case discussion contained four prompt categories: mathematical issues, analysis of student thinking, pedagogical issues, and contextual issues. Groups were required to select one choice from each category. One member of the group served as the recorder for each category, writing down as complete notes of the discussion as possible, including who made each contribution to the discussion. Initial concerns that the recording student might be less engaged in the discussion were generally unfounded. In fact, the recording group member often stopped the group discussion to catch up and then voice an opinion. Recording duties rotated with each category so that every group member served as recorder once during any case discussion.

The mathematical prompts for this case discussion were:

- What role do units and dimensions play in this problem? How well does each group of students understand the units and dimensions involved in the calculations and physical model? Explain.
- Answer the extra credit question posed by Mrs. Lister on page 32, right before the start of the "Lucy's Work" section. Which of the named students from this case do you think could solve this successfully? Which could not? Explain.
- Address Sergio's question from page 33. You may use a calculus argument for your own understanding, but also discuss what experiments could be done in Mrs. Lister's class to help her students reach an answer to Sergio's question.

The prompts for assessing student thinking were:

- LaShauna makes a comment on page 32 about the area of a square and a circle with the same perimeter. On that same page, Kelly makes a comment about pipes with different diameters. Both students are making connections to previous work. What are the similarities and differences in the prior learning they are trying to transfer?
- Analyze Lucy's work. What mistake(s) did she make? Why? What would you say to her?
- For each of the named students in the case, discuss what he or she learned or failed to learn about the relationship of perimeter and area to surface area and volume. Support your claims with evidence from the case.

The pedagogical prompts were:

- Did Heather ask the students to generalize too soon? Explain and support your answer with evidence from the case
- Are the manipulatives helpful in understanding and solving this problem? Are they always useful? Is hands-on learning always helpful? Support your answer with evidence not only from the case but also from your own experience as mathematics students and teachers.
- Heather was shocked to find out that her students did not understand the relationship between surface area and volume. She was also surprised to find out how tentative at least one student's understanding of exponents and square roots was. How might Heather have double-checked her assumptions about her students' understandings?

The contextual prompts were:

- The boys in this case seem to be much more confident than the girls. It also seems that they are more strident about being wrong, or not knowing how to solve the problem. Are the attitudes represented by these groups of boys and girls typical of what you may have observed in other math classes? What accounts for the differences? How do you think this affects attitudes and learning in mathematics?

- The seed of this idea was planted by a teacher colleague with whom Heather seems to have a good working relationship. What factors do you think help or hinder such collegial relationships between teachers? If you are a new teacher, how can you develop such relationships from the start?

2.5.2 Situated Practice

All of the PSTs were placed in cooperating classrooms and were able to situate a subset of the the case arcs and lesson study topics in practice in those classrooms. The four PSTs assisting in the Precalculus courses on the university campus, and all four of the GTA participants, were able to situate all of the case arcs and lesson study topics in practice within the Precalculus course. Since that situated practice was central to the structure of the teaching seminar and MATH 4080 course and thus to the design of this study, it is important to understand the structure of the Precalculus course itself.

The course is taken by STEM majors whose placement scores prohibit them from enrolling in Calculus I or Long Calculus A. The content covers essentially all of high school mathematics, from adding rational numbers through trigonometry and conic sections. The course structure is hybrid, with students working independently outside of class using an online learning and assessment program called ALEKS® (**A**ssessment and **LE**arning in **K**nowledge **S**paces) [ALEKS Corporation and McGraw-Hill Higher Educations, 2015]. Each face-to-face meeting has 60-70 undergraduate Precalculus students in two adjoining classrooms with one faculty member, two to three GTAs, and zero or one undergraduate CI assistants. Reporting features in ALEKS® allow for targeted individual instruction during the twice-weekly 75-minute face-to-face meetings. Students receive direct instruction only on topics they have attempted but been unable to master on their own using solely the online instructional materials. Direct instruction is to small groups where appropriate, and to individuals where warranted. Most direct instruction periods last 10 - 20 minutes within the 75-minute period. Precalculus students progress at their own pace and must complete all course objectives in order to pass the course. There are no scheduled exams covering fixed content; online assessment is ongoing.

2.5.3 Modified Lesson Study

In designing the first semester experience for the participants, we modified the traditional Japanese Lesson Study format for several reasons. From a purely pragmatic view, we couldn't do a multi-year refinement to create a "polished pearl" while situating the experience in a single semester seminar experience. Additionally, neither the GTA participants nor the PSTs had sufficient professional experience to undertake lesson study in its full form. However, a "polished pearl" wasn't the goal of the lesson study component. The intent, rather, was threefold:

1. To embed the GTAs in a teaching community of practice to foster aspects of "self-in-community" from Van Zoest & Bohl's model of teacher identity to development.
2. To provide a shared situated practice experience that would promote development into the pedagogical content knowledge side of Ball et al.'s model of teacher identity.
3. To provide a setting in which the GTAs could try out a provisional self with a strong teacher identity within Ronfeldt & Grossman's model of professional identity development.

This experience needed to fit within the time frame of the semester and within the content range of the Precalculus course. We therefore used a modified form of Japanese Lesson Study in which teams selected a topic, researched the topic, planned a short lesson, delivered that lesson in the Precalculus class, reflected on the delivery, and revised the lesson [Allvine et al., 2007, Gorman et al., 2010]. There were two cycles of lesson study, with teams changed between each cycle. Teams were constructed to include one GTA and four undergraduates. Teams selected from a menu of eight topic options each cycle. The first cycle consisted primarily of topics from the algebra curriculum. The second cycle consisted of topics from trigonometry. All menu topics have historically required direct instruction to small groups in the Precalculus course. See Table 2.5.2 for a complete list of the eight choices for each cycle.

Cycle	Topics
1	✓Graphing piecewise defined functions Converting between quadratic form and parabolic form Graphing rational functions (no formal limit arguments, but deal with asymptotes) ✓Solving polynomial inequalities (graphically and algebraically) ✓Solving word problems to find the maximum or minimum of a quadratic equation Solving absolute value inequalities ✓Finding the domain for the composition of two functions ✓Determining the inverse of a function (graphically and algebraically)
2	Coordinates of special points on the unit circle ✓Locating terminal sides of special angles ✓Using the $(x, y) = (\cos\theta, \sin\theta)$ relationship to determine trig ratios Using generalized trig ratios (e.g. $\sin\theta = y/r$) to determine trig ratios ✓Transformations of sine and cosine graphs ✓Solving trig equations using the unit circle to get exact value solutions Inverse trig functions (domains and ranges, evaluation) ✓Law of Sines and Law of Cosines to solve triangles, including the ambiguous cases

Table 2.5.2: Topics Available and Selected for the Lesson Study Cycles. Topics selected by the teams are indicated by a checkmark (✓).

2.6 Second Semester and Second Year Experiences

During the second semester, one participant continued with an assignment to Precalculus. Two were GTA's for other lower-division courses. One was assigned as a teacher of record for one 45-student section of Business Calculus I. None participated in a regular, organized discussion of teaching mathematics. During their second year in the graduate program, three of the participants were assigned as teachers of record for undergraduate courses. Two taught a single 45-student section of Long Calculus I. One taught three 19-student sections of Business Calculus I. The fourth was placed on research assistantship.

2.7 Reliability and Validity

Issues related to reliability (accurately measuring and interpreting responses) and validity (actually measuring what we intended to measure) for individual data collections and analysis are addressed within the relevant sections of Chapters 3 and 4. However, we note again that the intent of the research design was to draw from best practices in K-12 teacher education to provide, within the framework of current graduate school practice, the greatest support possible for development of a robust teacher identity within the overall academic mathematician identity. We are not attempting to assess the effectiveness of any single intervention. Rather, each piece of data is intended to measure the participant's trajectory within the teacher identity framework, teacher identity development framework, and/or professional identity development framework. It is ultimately the reliability and validity of the data as it is used to address the research questions that is at issue, and it is at that level that triangulation between multiple measures is most powerful.

Having multiple measures and types of data collected at multiple time points lends strength to reliability. Although we are trying to promote growth in teacher identity over time, it is unrealistic to expect a complete change of identity in a span of nine months. We therefore expect to see consistency in key aspects of identity over time and across measures. Where we see marked change or conflicting measures, we seek explanatory evidence or question the reliability of the conclusion. Issues of reliability are most relevant at the individual case level, and are revisited within each individual case analysis in Chapter 5.

The use of multiple frameworks to analyze the bulk of the qualitative data lends strength to the validity of the data. The aspects of teacher identity considered in Beijaard et al.'s model of teacher identity through a personal knowledge perspective should theoretically align at key points in Van Zoest & Bohl's model of teacher identity development. Similarly, aspects of self-in-community from Van Zoest & Bohl's model should theoretically align with key aspects of Ronfeldt & Grossman's model of professional identity development. Data that do not align are worthy of deeper examination, and are addressed in Section 4.6. Where the data do align as anticipated within these frameworks, it lends considerable validity to our conclusions. Issues of validity are most relevant at the collective data level and are revisited in both in the qualitative data analysis in Chapter 4 and in the cross-case analysis in Chapter 6.

2.8 Researcher Bias

The researcher served as both the instructor of record for the seminar course and as supervising faculty for the Precalculus teaching assignment for the study participants. One of the study participants continued with the researcher as supervising faculty for a second semester. The researcher's views on teaching and on professional practice formed, by design, an integral part of the teaching seminar and the classroom practice. Thus the specter of researcher bias in interpretation and analysis of the data looms large. While we made no attempts whatsoever to minimize researcher *impact* on the participants' views of teaching and professional practice, we took as many precautions as were reasonable to minimize potential researcher *bias* in the data analysis portion of this study.

Surveys were administered in class, but students were informed that they were for a completion grade and would not be reviewed until after the end of the semester. Written reflection assignments were administered under the same instructions. Interviews were conducted by two of the researcher's colleagues who had no other form of contact with the participants. Lesson study and case arc artifacts were, by necessity, made available to the researcher for grading purposes during the first semester. In-depth analysis of those artifacts using the theoretical frameworks on which the study was built did not occur until after the conclusion of the second semester activities.

The results of the pre- and post-surveys, reflective writings, and interviews were sealed until after the conclusion of the first semester activities. The first interviews were reviewed briefly during the second semester in order to structure the second interview protocols. In-depth analysis of the reflective writings and interviews, however, was postponed until after the conclusion of the second semester since one of the participants was still actively engaged in teaching practice with the researcher. After the conclusion of the second semester, and throughout the analysis of the data, the researcher had no contact with the study participants beyond occasional greetings in the mailroom.

Prior to analysis of the qualitative data, the researcher responded to bracketing prompts to identify and mitigate potential researcher bias. Each bracketing response was approximately two paragraphs. After a one week waiting period, the researcher then reviewed her own responses to observe themes that might indicate bias and developed a strategy for mitigation of that bias. The prompts for the bracketing are provided in Appendix A. The results of the bracketing are discussed in more depth in the sections of Chapter 4 dealing with analysis of the qualitative data. The importance of that bracketing process cannot be overstated; it proved critical in identifying and mitigating potential bias in the data analysis and conclusions.

Chapter 3

Quantitative Data Analysis

This section addresses the two quantitative data collections as separate entities, including type and time of data collection, how the data was analyzed, and results from that data collection in isolation for all four participants combined. Both sections in this chapter are included nearly verbatim from a published paper [Gallagher et al., 2016]. Modifications to the published paper appear in italics. However, since the individual case analyses and cross-case analysis for the dissertation include more extensive sources of data than were used in that paper, and thus are more robust, that paper as a whole is not included as its own chapter in this dissertation. See Chapters 5 and 6 for the integration of the quantitative and qualitative data to address the research questions.

3.1 Survey

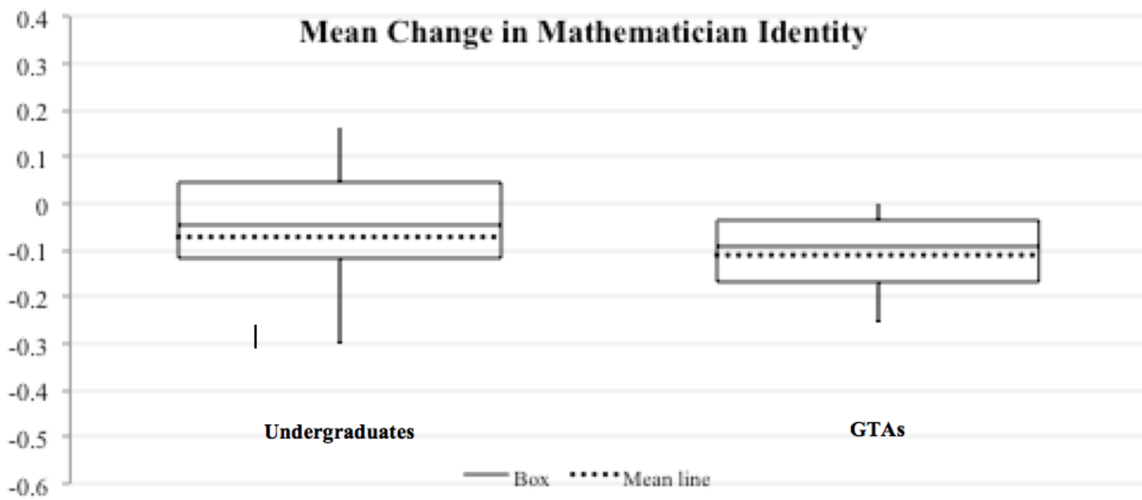
All 23 of the students in the combined course provided survey responses and course artifacts used in the study. On the first and last day of the combined course, students completed a 101-item survey using forced-choice Likert-like items. The four response categories were "strongly agree," "agree," "disagree," and "strongly disagree." Each category score was translated to a numerical score of "1" for "strongly agree" up to "4" for "strongly disagree". For positively correlated items, score change was calculated as pre-test score minus post-test score. Negatively correlated items were reversed: score change was calculated as post-test score minus pre-test score. Thus, for a positively correlated item such as "College math professors are expected to teach well" a student who agreed (2) on the pre-test but strongly disagreed (4) on the post-test would have a score change of -2. The same pre/post responses for a negatively correlated item such as "I would like a job in which I don't have to teach" would receive a score change of $+2$.

The items were divided among three subscales:

- Mathematician identity (43 items). These questions are intended to elicit the extent to which the participant identifies as a mathematician either currently or as a future goal. The items in this subscale were closely adapted from the Mathematics Attitude Inventory [Welch, 1972] (MAI) and loosely adapted from FICSMath [Sonnert, 2009]. The MAI was previously validated for preservice elementary mathematics teachers; items were modified to reflect secondary and post-secondary topics. FICSMath was validated for use with college freshmen across disciplines [Cribbs et al., 2015]. Positive score change within this subscale indicates a movement towards a stronger view of oneself as a mathematician.
- Epistemological beliefs and attitudes (25 items). The items in this subscale are intended to elicit participants' views on the nature of mathematics and knowledge through the lens of views on effective mathematics instruction. These items were taken from a survey instrument previously validated for preservice elementary mathematics teachers [Roberts, 1993] and modified to reflect secondary and post-secondary topics. Positive score change within this subscale indicates a movement towards a more constructivist view of mathematics and the teaching of mathematics, i.e. towards a belief that students can construct their own understandings of mathematics and that effective instruction capitalizes on that.
- Teacher identity (33 items). The items in this subscale are intended to elicit the extent to which the participant identifies as a teacher, either currently or as a future goal. As with the "mathematician identity" subscale, these items were adapted from the MAI and from FICSMath. Positive score change within this subscale indicates a movement towards a stronger view of oneself as a teacher of mathematics.

Each of surveys from which items were taken intact or adapted slightly had previously undergone test-retest validation. However, the source instruments were validated for populations other than the subject population, so drawing conclusions about identity or epistemological beliefs from a single survey administration alone should be taken as tentative. An item-by-item change analysis is beyond the scope of this paper; we include only results for each subscale as a whole. These results are intended to provide general insight into shifts over time, rough comparison between the two populations, and triangulation with case artifacts and student performance data.

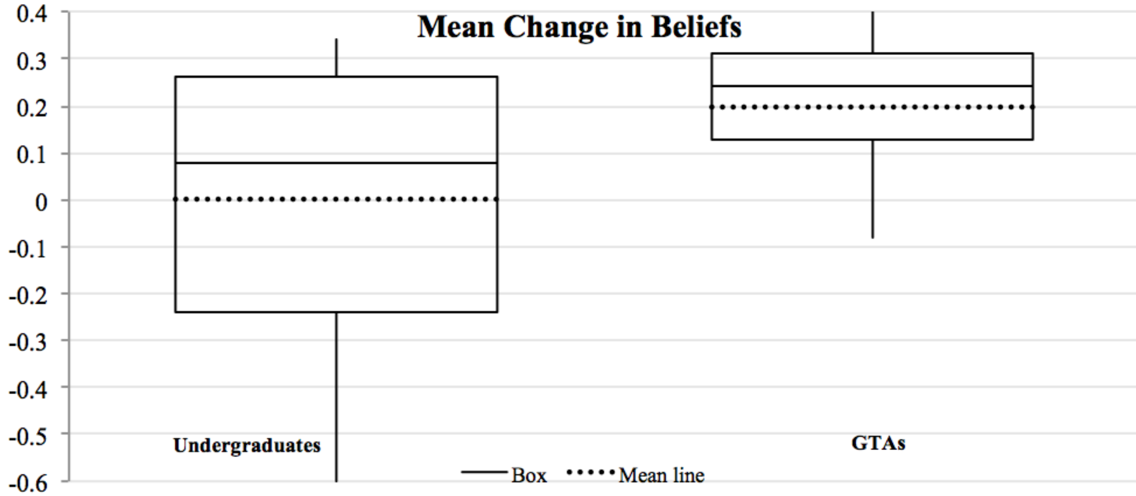
We collected pre- and post-survey data for all four GTA participants and for 17 of the 18 undergraduate pre-service teachers. In the absence of validation for the GTA population, we cannot make assumptions of normality to allow for a two-sample t-test comparing mean changes between the two *groups*. Nor, given the small sample size for the GTA *sample*, can we use a two-sample z-test. Thus, we cannot test for statistically significant differences between the mean changes in subscale score for the two *groups*. However, we can make some observations using descriptive statistics, so we present here the box-and-whisker plots for mean change in each subscale for both *groups*.



Population	*Count*	*Mean*	*Minimum*	*Lower whisker*	*Q1*	*Median*	*Q3*	*Upper whisker*	*Maximum*
Undergraduates	17	-0.06977	-0.30233	-0.30233	-0.11628	-0.04651	0.04651	0.16279	0.16279
GTAs	4	-0.11047	-0.25581	-0.25581	-0.1686	-0.09302	-0.03488	0.	0.

Figure 3.1.1: Changes in Mathematician Identity as Measured on Pre-Post Survey. A positive change indicates a shift towards a stronger mathematician identity.

Both populations were taking mathematics courses at a higher level than in their previous experiences. All of the undergraduates were taking either abstract algebra or advanced calculus for the first time; some were taking both. The graduate students were taking graduate level mathematics courses for the first time. Although the mean changes between the two populations are close, we note in Figure 3.1.1 that while somewhat more than a quarter of the undergraduates had a shift towards a stronger mathematician identity, none of the graduate students did.

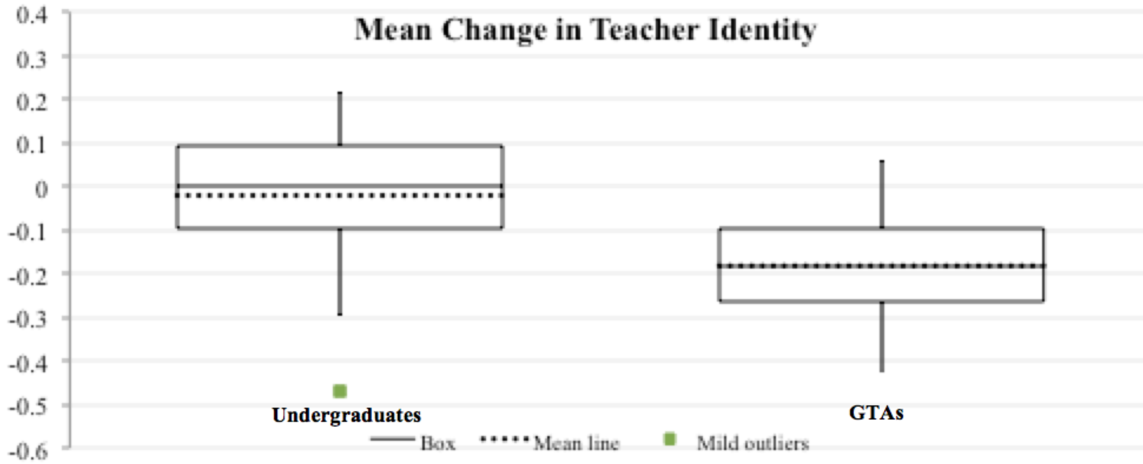


Population	*Count*	*Mean*	*Minimum*	*Lower whisker*	*Q1*	*Median*	*Q3*	*Upper whisker*	*Maximum*
Undergraduates	17	0.00118	-0.6	-0.6	-0.24	0.08	0.26	0.34	0.34
GTAs	4	0.2	-0.08	-0.08	0.13	0.24	0.31	0.4	0.4

Figure 3.1.2: Changes in Epistemological Beliefs as Measured on Pre-Post Survey. A positive change indicates a shift towards a more constructivist view of mathematics.

The undergraduate students had previous experience in secondary mathematics education coursework presented with a constructivist view. Their mean change in epistemological beliefs was lower than that of the GTAs, as shown in Figure 3.1.2. In addition, 75% of the GTAs displayed a greater shift towards a constructivist view of mathematics and teaching than the mean and median change for the undergraduates.

It is in the teacher identity subscale that we see the most striking difference between the two populations. The undergraduates had a median change of zero, indicating no change in their teacher identity. The data for the undergraduates is nearly symmetric, with one slight outlier dragging the mean slightly below the median. The GTAs demonstrated a striking shift *away* from a teacher identity, with their entire interquartile range falling in the lowest quartile of the undergraduate population. *Consideration* of this shift within the context of case arcs and reflective writings presents a more complete picture for each individual, and is addressed in the discussion section.



Population	*Count*	*Mean*	*Minimum*	*Lower whisker*	*Q1*	*Median*	*Q3*	*Upper whisker*	*Maximum*
Undergraduates	17	-0.0193	-0.46875	-0.29688	-0.09375	0.	0.09375	0.21875	0.21875
GTAs	4	-0.18182	-0.42424	-0.42424	-0.26515	-0.18182	-0.09848	0.06061	0.06061

Figure 3.1.3: Changes in Teacher Identity as Measured on Pre-Post Survey. A positive change indicates a shift towards a stronger teacher identity.

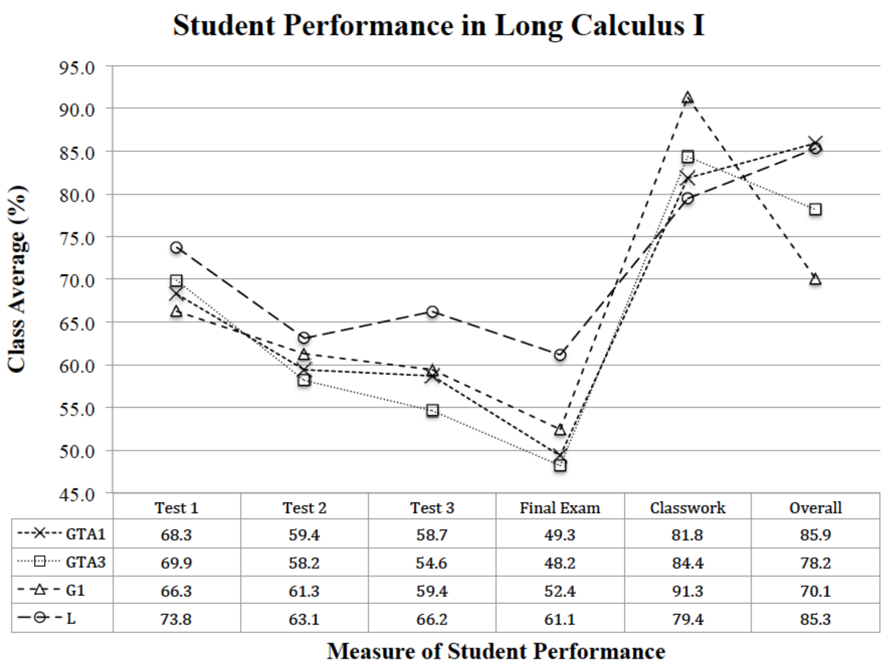
3.2 Student Performance

All introductory mathematics courses at the participant institution are closely coordinated, with common midterm and final exams. We compare student performance in the participants' sections to student performance in sections taught by other GTAs in their first teaching experience, as well as to overall student performance in all sections. This quantitative data contributes to triangulation with respect to participants' self-perception of teaching competence. It also provides a small degree of insight into effectiveness of the use of case study for professional preparation during the first year of graduate studies.

There are *concerns* regarding the fidelity of the data for coordinated Business Calculus course during the first semester as instructor of record for one of the subjects. A second subject was placed on research assistantship, so we have no data for student performance for that subject.

The remaining two subjects, GTA1 and GTA3, were each assigned to teach one section of Long Calculus I with 45 students each. The content of Long Calculus I is one-third review of precalculus, one-third limits, and one-third introduction to derivatives (up to the chain rule). The course is closely coordinated, with common online homework, midterm exams, and final exam. Exam grading is done collectively, with one grader for the same problem across all sections. The didactical and pedagogical aspects are at each instructor's discretion, and there is no ongoing instructional support provided to GTAs.

Results from the participant sections and from comparison sections are given below in Figure 3.2.1. In this figure, GTA1 and GTA3 are study participants, G1 is a first-semester teacher of record who had not participated in the study activities and L is the average of two experienced full-time lecturers teaching six sections total. Test 1, Test 2, Test 3, and Final Exam are the collectively graded common exams. "Classwork" is the portion of the course grade that is instructor-dependent and based on classroom instructional activities. "Overall" refers to the final weighted course average. It is worth noting that this is a Pass/Fail course that does not affect overall GPA, and students with sufficiently high course averages are exempt from the final. Both of those factors depress final exam averages, particularly in sections with stronger performance prior to the final.



	Test 1	Test 2	Test 3	Final Exam	Classwork	Overall
GTA1	68.3	59.4	58.7	49.3	81.8	85.9
GTA3	69.9	58.2	54.6	48.2	84.4	78.2
G1	66.3	61.3	59.4	52.4	91.3	70.1
L	73.8	63.1	66.2	61.1	79.4	85.3

Figure 3.2.1: Comparison of Student Performance in Subsequent Teaching Experiences. 'G1' indicates first-time graduate teachers of record who were not study participants. GTA1 and GTA3 were study participants in their first teaching assignment, and 'L' indicates experienced full-time lecturers teaching the course.

On each of the common exams, the three first-time teachers of record appear to be closely matched, and student performance falls below that of experienced lecturers. Final course averages were significantly higher for the GTAs who participated in the combined course that for non-participants, however, despite the fact that they assigned lower daily grades, the only portion of the course average under their direct control. While this superficially appears contradictory, it actually reflects declining student participation in the section taught by G1. Students who did not take a given exam are not reflected in that exam average but are reflected in the course average, so as some students stopped attempting exams, the exam average for that section increased, but the overall course average dropped. Thus, it can be argued that GTA1 and GTA3 were more successful at student retention and engagement over the course of their first solo teaching experience than was G1. This may in turn reflect stronger ability to engage students as learners, one aspect of pedagogical expertise. It is also worth noting that the overall student performance in GTA1's section matched that of the experienced lecturers. *This reflects the fact that the pass/fail grading structure and ability to exempt out of the final exam may depress final exam average in a section with students who are doing well in the course. Under these circumstances, overall course average at the end of the term is a more accurate reflection of student performance than any single measure during the semester.*

Chapter 4

Qualitative Data Analysis

Analysis of the case artifacts is addressed in Section 4.1 and appears nearly verbatim from a published paper [Gallagher et al., 2016]. Modifications to the published version appears in italics. The interviews, responses to reflective writing prompts, and written reflections from the lesson study were analyzed using similar methods, described in Section 4.2 of this chapter. Results from these data are analyzed as an integrated unit for all four participants combined in Sections 4.3, 4.4, and 4.5 and compared across frameworks in Section 4.6. The more interesting question, of course, is how the data was integrated to address the research questions. That integration and analysis is covered in the individual case analyses in Chapter 5, and also in the cross-case analysis in Chapter 6.

4.1 Case Artifacts

Analysis of the case artifacts is based on Beijaard, Verloop, and Vermunt's model of teacher identity from a personal knowledge perspective [Beijaard et al., 2000]. They divide teacher identity into three poles as shown in Figure 4.1.1. Subject-matter expertise is knowledge of the content of the course. Didactical expertise is the ability to plan, conduct, and assess a class session. Pedagogical expertise in this context includes such aspects as supporting the psychological and emotional well-being of the students, engaging students in the learning process, and adapting to meet the needs of individual learners. Individuals are located within the framework based on the relative importance they assign to each of these types of expertise.

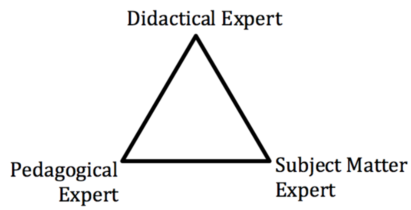


Figure 4.1.1: Beijaard, Verloop, and Vermunt's model of teacher identity through a personal knowledge perspective.

Early career secondary mathematics teachers typically cluster along the pedagogical/didactical axis, with experienced secondary teachers moving towards the center of the triangle [Beijaard et al., 2000]. Mathematics graduate programs traditionally take the implicit view that a subject-matter expert with minimal on-the-job training will be an effective teacher [Burton, 1999, Hirt and Muffo, 1998]. To the extent that additional training is provided for mathematics GTAs, that training falls largely in the didactical branch of this model [Cox et al., 2009, Speer et al., 2005, Park, 2004]. Thus, most research mathematicians and mathematics GTAs likely fall along the subject-matter/didactical edge of Beijaard's triangular identity model.

Rather than ask participants to rank-order statements that might be construed as aligning with a particular expertise, we coded their comments during post-case discussions as being aligned with one of the three poles: subject-matter, didactical, or pedagogical. See Excerpt 4.1.2 for an excerpt from one case discussion and how comments were coded; this excerpt is in response to the second "assessing student thinking" prompt from the Slippery Cylinders case.

The relative proportion of each type of comment situates the discussion within the framework during each of the case arcs. In particular, it provides a visual representation for each participant of their movement (or lack thereof) towards the more central location occupied by experienced secondary mathematics teachers. To clarify how each participant was located in the framework based on their comments during a single case discussion, we provide an example. Suppose the count was six "subject-matter" orientation (S) comments, three "didactical orientation" (D) comments, and two "pedagogical orientation" (P) comments. Comparing categories S and D, 6 of the 9 comments were in category S, so we place a mark 2/3 of the way along the SD side of the triangle, closer to S. Similarly, we place a mark 3/4 of the way along the SP side of the triangle (closer to S) and a

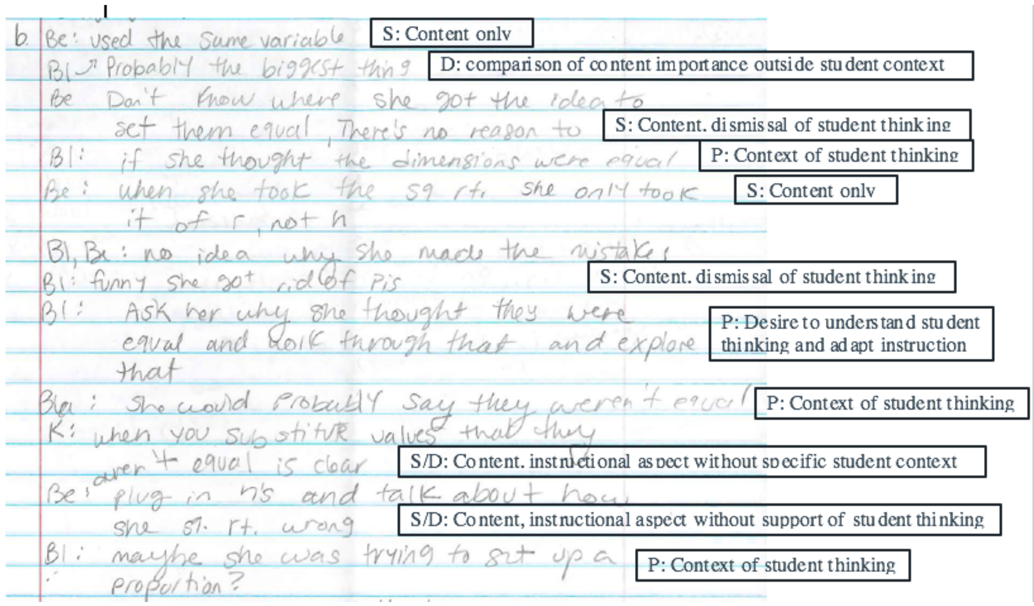


Figure 4.1.2: Sample of case discussion record from Slippery Cylinders, together with coding. "S" indicates subject-matter expertise, "D" indicates didactical expertise, and "P" indicates pedagogical expertise.

mark 3/5 of the way along the DP side of the triangle (closer to D). We then construct the triangle created by those three marks. The participant's location within the framework is at the incenter of the constructed triangle, where the three angle bisectors meet (see Figure 4.1.3). *We note that it is the structured nature of the prompts that allows us to quantify what is inherently qualitative data. The individual comments reflected similar depth and thus can be considered to have similar weight. Moreover, since the prompts were designed to elicit comments evenly from each category, the relative proportion of comments is a reasonable proxy for identity location.*

It is within the framework of the discussion prompts that we analyze the identity locations of each participant and construct an identity path. As noted, we did not reach a full case discussion for Ships in the Fog. The identity paths for the remaining six cases for each of the four GTA participants are shown in Figure 4.1.4. It is important to note that these are extracted comments from group discussions. As such, they represent only a fragment of the entire discussion in which the subject was involved for each case, but they critically represent the areas in which the participant was willing to voice his or her view and make a contribution. Thus, they provide interesting insight into the "comfort zone" of each participant over the duration of the combined course. In contrast, a similar construction for one of the senior mathematics education majors is shown in Figure 4.1.5.

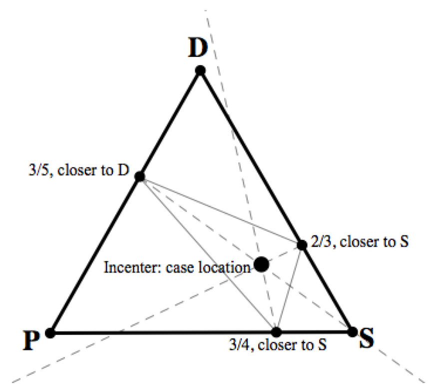


Figure 4.1.3: Adaptation of Beijaard, Verloop, and Vermunt's Model for Teacher Identity. Location of one participant during one case discussion based on the relative number of comments within each category. "S" indicates subject-matter expertise, "D" indicates didactical expertise, and "P" indicates pedagogical expertise.

For brevity, we do not include the other 17 identity paths, but the one shown is representative of the remaining undergraduates.

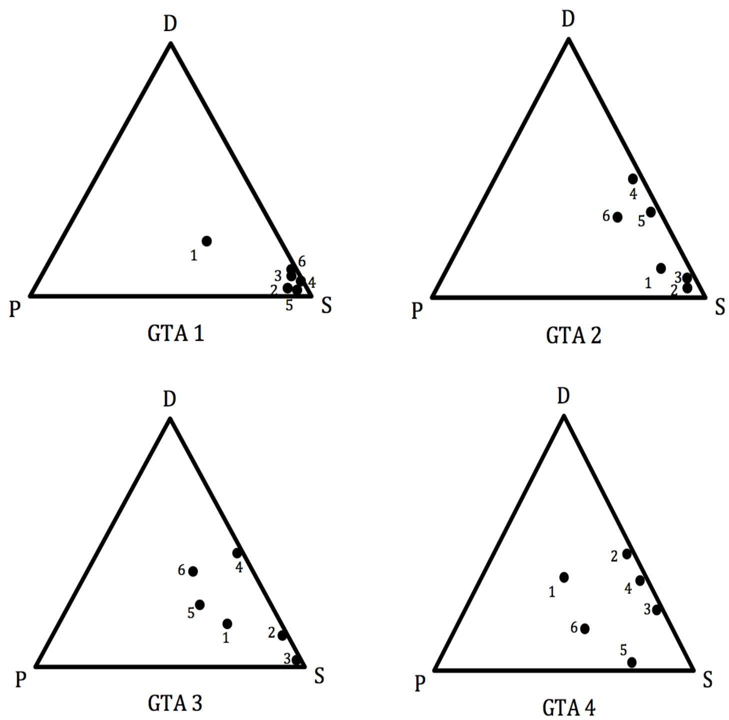


Figure 4.1.4: Graduate Student Teacher Identity Trajectories Based on Case Artifacts. The identity paths of the four GTA participants in the combined course, based on comments in case discussions. "S" indicates subject-matter expertise, "D" indicates didactical expertise, and "P" indicates pedagogical expertise.

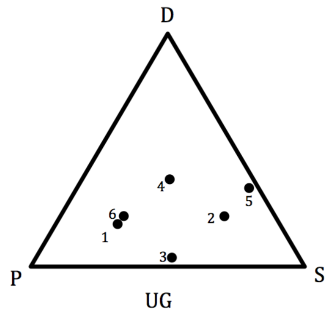


Figure 4.1.5: Sample Undergraduate Teacher Identity Trajectory. The identity path of a typical undergraduate mathematics education major in the combined course, based on comments in case discussions. "S" indicates subject-matter expertise, "D" indicates didactical expertise, and "P" indicates pedagogical expertise.

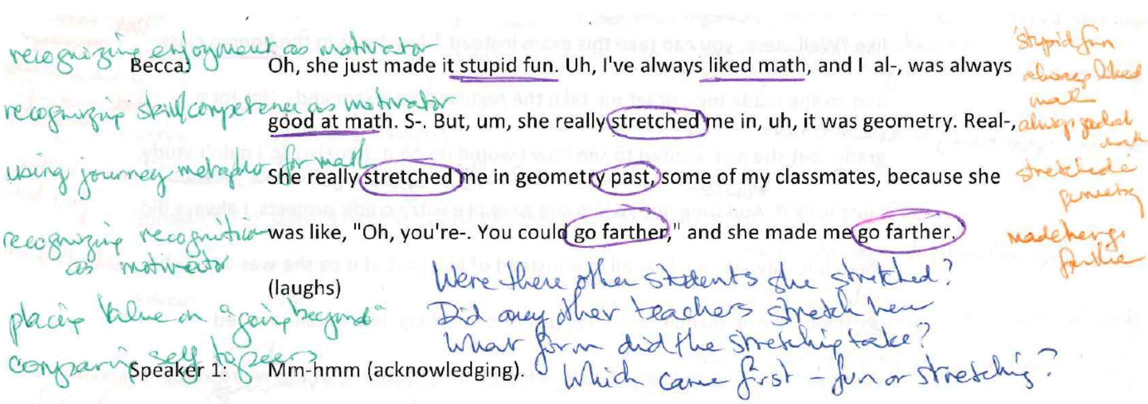
4.2 Coding Process for Interviews and Written Reflections

The GTAs were interviewed Week 11 of the first semester and again at the end of their second semester in graduate school. The interviews were loosely structured and lasted about 30 minutes each. The first round of interviews had the same eight initial prompts (see Appendix D) for each of the four participants with follow-up questions by the interviewers to probe more deeply where needed. Note that the first interview also contained a prompt about teaching a specific mathematical topic in order to elicit views on teaching through a hypothetical situation. The second round of interviews had nine prompts for each participant (see Appendix E for one example). The prompts were on similar themes, but were individualized to include details from the previous interview. The second interview focused on possible selves and on identifying current factors influencing view of future self. Interviews were transcribed verbatim, including filler words, from audio recordings by a secure third-party service.

Each participant responded to written prompts after the first lesson study cycle and at the conclusion of the course. The prompts for the written reflections are included in Appendix C. Each prompt was provided on a blank sheet of paper and the prompts were distributed one at a time. Participants were given 15-20 minutes to respond to each prompt. At the conclusion of the allotted time, participants submitted the written response and received the next prompt. Order of the prompts was randomized for each participant.

The two lesson study cycles each included written reflections after both lesson deliveries, so we should have a total of four written reflections per participant. However, some of the participants chose not to complete one or more of the written reflections on the lesson study observations. We therefore have only 11 lesson study reflections, rather than the full set of 16. The requirements for the written reflections were a) that they be completed within 24 hours of observation of the lesson delivery, 2) that they be 1-2 pages in length, 3) that they address aspects of the lesson that met or failed to meet the team's goals and expectations for the lesson, and 4) that they include recommendations for improvement of the lesson.

The interviews and both types of written reflections were coded through an interpretive phenomenological lens in several stages. The first stage involved revoicing key passages related to participant experience. The second stage was a linguistic analysis, looking for patterns of language usage that might illuminate themes in how the participant internalized his or her experiences. The third stage was to identify additional questions raised by the responses of the participant. The fourth stage involved writing preliminary emergent codes that reflected the essence of the experience as it was lived by the participant. A sample of one interview is included in Excerpt 4.2.1 with the multiple stages of coding for elucidation of the process. The process for the written reflections was similar, although not as rich given the brevity of the responses and the lack of opportunity to probe for depth.


recognizing enjoyment as motivator
recognizing skill/competence as motivator
using journey metaphor for math
recognizing recognition as motivator
placing value on "going beyond"
comparing self to peers

Becca: Oh, she just made it stupid fun. Uh, I've always liked math, and I al-, was always good at math. S-. But, um, she really stretched me in, uh, it was geometry. Real-, She really stretched me in geometry past, some of my classmates, because she was like, "Oh, you're-. You could go farther," and she made me go farther.

(laughs)

Speaker 1: Mm-hmm (acknowledging).

Were there other students she stretched?
Did any other teachers stretch her?
What form did the stretching take?
Which came first – fun or stretching?

stupid fun
always liked math
always good at math
stretched in geometry
made her go further

Figure 4.2.1: Interview Excerpt Showing All Four Coding Stages. Excerpted from Interview 1 with GTA3. Orange comments in the right column reflect Stage 1 analysis (revoicing). Purple marks within the text reflect Stage 2 analysis (linguistic). Blue comments above or below the excerpt text reflect Stage 3 (questions raised by the excerpt). Green text in the left margin indicates the initial emergent codes assigned to the excerpt.

Between interviews, reflection prompt responses, and lesson study reflections, we have a total of

43 files containing qualitative data from the four participants. All files, codes, and preliminary emergenct codings were entered into RQDA [R Core Team, 2016] for category groupings and comparative analysis. After entering the preliminary coding, those initial emergent codes were reviewed to identify redundancies. In many cases, the initial phrasing of the code was revised for clarity where revision did not dilute the essence of the code and/or where it reflected similar experiences as a differently phrased code. For example, the initial codes "Handling teaching discomfort by looking online for suggestions" and "Using online resources to develop instructional strategies" were combined and refined to "Developing instructional strategies: online".

Following integration and subsequent refinement of codes from multiple sources, codes were reviewed to identify thematic groupings. The groupings were done in three separate reviews: once each for themes related to Beijaard et.al's model of teacher identity (see Figure 4.3.1), Van Zoest & Bohl's model of teacher identity development (see Figure 4.4.1), and Ronfeldt & Grossman's model of professional identity development (see Figure 4.5.1). A table of the code categories, identifying characteristics, and an illustrative sample of each category is provided in the sections on analysis according to each of the three frameworks. A complete listing of all codes and how they were assigned within each of the three frameworks is found in Appendix F. Codes that fit into multiple categories within a given framework are discussed within the pertinent section, as are examples of excerpts that were assigned multiple codes. In Section 4.6 we give examples of how some sample excerpts fit into each of these three frameworks in different ways, and a discussion of how the three frameworks aligned and ways in which they differed.

To clarify the language used in the following sections, we note that "code" refers to a descriptive phrase and "coding" refers to an excerpt from the data together with its code, whereas "code category" or just "category" refer to a collection of codes. A single excerpt may be assigned different codes; each of those results in a different coding. A single code may be assigned to multiple excerpts; each of those also results in a different coding. A single code may have codings pulled from multiple data sources and participants. The same code may be assigned to multiple categories, both within the same framework and across frameworks. Assignment to a new category does not change the codings included within the code. Unless otherwise indicated, "the data" refers to both interviews, all six written reflection prompts, and all four lesson study reflections taken in aggregate.

4.3 Analysis Using Beijaard et al.'s Framework

For easy reference, we present again Beijaard et al.'s model of teacher identity.

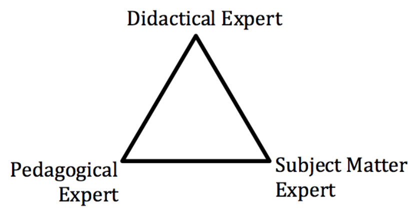


Figure 4.3.1: Beijaard, Verloop, and Vermunt's model of teacher identity through a personal knowledge perspective. "S" indicates subject-matter expertise, "D" indicates didactical expertise, and "P" indicates pedagogical expertise.

For this analysis, we focused solely on portions of the data that pertained to teaching mathematics. After refining the initial coding of excerpts, codes were classified as representing a Subject Matter Orientation, Didactical Orientation, or Pedagogical Orientation, as described in Table 4.3.1. We note that for this model, we are using the classifications as Beijaard et al. described them, but with a view to *orientation* rather than *expertise*. We cannot assume that each coding from the interviews and reflective writings is of similar importance and thus we do not perform the same quantification based on relative counts as we did in analyzing the case artifacts.

Code Categories within Beijaard et al.'s Framework of Teacher Identity		
Category	**Criteria**	**Sample Excerpt**
Didactical Orientation	Planning and executing a lesson, structuring time and activities within the class period, assessing the lesson and outcomes including student mastery	"When I graphed the parabola, I should have asked them what it would look like (downwards, with the y-intercept of zero), and I should have asked them how to label the axes. Basically, I could have asked them even more questions than I did; more questions would have helped me know if their prior knowledge was where it should have been. I think I had good wait time after questions and that I scaffolded well when they were unable to answer the question as I phrased it."

Continued on next page

Table 4.3.1 – continued from previous page		
Category	**Criteria**	**Sample Excerpt**
Pedagogical Orientation	Supporting the emotional and social needs of the students, adjusting instruction to meet individual learning needs	"Just trying to understand how your students think through something, um, because everyone learns differently, everyone sees things differently when they read something for the first time. Everyone will kind of interpret it, uh, slightly differently. Yes, they'll get the-, they might get the meaning but they'll come about that meaning in their own way."
Subject-Matter Orientation	Addressing the mathematical content of the lesson and/or the participant's understanding of that content (as opposed to student understanding of that content)	"Oh, if you're going to graph it? I guess, assuming that they know how to graph lines I would tell them you graph it as a line and then you make it dotted if it's strict inequality and straight if it's weak inequality. And then just make a point on either side of the line and test if and if it makes the relationship true, then shade the side with the point on it. And if it gives you a false dimension you shade the other side."

Table 4.3.1: Qualitative Data Code Categories for Beijaard et al.'s Framework. Code categories adapted from Beijaard et al.'s framework for teacher identity, together with criteria for inclusion in a category and samples for each code category.

The pre-analysis bracketing proved critical in reducing researcher bias in assigning codes to categories within each framework. For example, Excerpt 4.3.1 was coded as "Valuing patience".

> Int: When we talked in October you identified patience as one of the most critical skills to develop to be successful as a math teacher. Is that, a key thing still?
>
> GTA2: I think it's still the same. Umm, to expand on that though, one thing that I've found that, umm, I've learned, I picked up from [supervising teacher] a bit, is, the patience to ask a question and sit there silently for five or ten seconds, or more, maybe. Or, and then try and ask it again, instead of just asking a question and no one responds right away, so, just move on from it. Umm, that's a, definitely a hard thing for me because I do get nervous public speaking and when I'm nervous, I tend to talk really fast.
>
> Int: Mm-hmm (affirmative).
>
> GTA2: And so I have to slow myself down just in general. And then when I sit there in silence, it feels like ten minutes and it's been three seconds, then I'm like, "All right, yeah, let's just keep going then. No one's going to answer, so let's keep moving on."
>
> Int: Mm-hmm (affirmative).
>
> GTA2: And they are, they're not even where I'm at yet, so.
>
> Int: It seems like an eternity when it's just, yeah.
>
> GTA2: Right, yeah. So I would say patience still, but I guess, kind of like a different aspect, maybe.

Excerpt 4.3.1

The researcher initially assigned Excerpt 4.3.1 to the Pedagogical Orientation category within Beijaard's framework as a result of bias both towards use of wait time and towards the word 'pedagogical' as connoting higher value than the word 'didactical'. Ultimately, however, wait time is a didactical tool: one choice of method of instruction. The excerpt itself does not explicitly address the emotional or social needs of the students, nor does it discuss adjusting instruction to meet individual learner needs. Review of the initial assignment of codes to categories side-by-side with the bracketing analysis eventually led to the code "Valuing patience" being assigned to the Didactical Orientation category within Beijaard's framework. Note that this excerpt was also assigned the code "Using resources:instruction:experienced teachers" but that code was not assigned to a category within Beijaard's framework.

The Didactical Orientation category also included codes such as "Reflecting on practice: didactical basics" which included codings dealing with boardwork, speaking volume, and basic classroom management. Excerpt 4.3.2 provides a sample coding from that code.

> "We had to give a few, um, practice lectures and that was really helpful because you got feedback from, you know, the audience as to you need to write bigger on the board or talk slower or this example didn't quite fit with what you said or stuff like that to help you kind of think about how ... Not so much how to plan the lecture but how to deliver it so that it's understandable. Um, and that was really good."

Excerpt 4.3.2

In order for a code to be grouped within Pedagogical Orientation, the excerpts associated with that code had to reflect concern with the emotional and social needs of the students, and/or adjusting instruction to meet individual learning needs. For example, Excerpt 4.3.3 was coded as "Handling student confusion: focus on student affect" and grouped within Pedagogical Orientation.

> "The one thing I would watch for is frustration in the student. I have found that when a student gets frustrated, they shut down and pretend to understand, which is never good."

Excerpt 4.3.3

In the Subject Matter Orientation category, we placed codes that related to the mathematical content of the lesson or the participant's understanding of that content (including gaps in their understanding) or to a direct statement about relative importance of expertise for teaching. The excerpt included in Table 4.3.1 for this category is a sample focused on the mathematical content. Excerpt 4.3.4, on the other hand, was coded as "Valuing content knowledge:teaching" and demonstrates the third type of criteria for inclusion in the Subject Matter Orientation category.

> Int: So do you still agree with the idea that, um, knowing the material really well, keeping sight of where the students are is the most important?
>
> GTA1: Um, I guess. I mean I don't know if it's ... It's definitely not enough, it's not the only thing you need to do but I think if you don't know the material, nothing else ... I mean, there's no way to overcome that so I guess it's most important in the sense of, if you don't have that you're completely out of luck.

Excerpt 4.3.4

Not all of the codes associated with a category reflect a positive orientation within that category. For example, within the Pedagogical Orientation category, we have the code "Distancing self from students," exemplified in Excerpt 4.3.5. It shows a surface desire to understand the students, but also an underlying attitude that because the GTA's background was very different than that of her students, she would be unable to understand and help them. Nonetheless, because the excerpts in this code reflect *awareness* of individual learner needs, despite either an unwillingness or an inability to adapt instruction to meet those needs, this code was included in Pedagogical Orientation.

> GTA3: "Um, one of the things I struggled with most while I was student teaching was to try and understand my students, because I had all the really low-level students, and they didn't get it, and they didn't get it, no matter what I did. It was like, 'Why won't you get it?' That's just because, because my background and my mathematics was so much different than theirs.
>
> Int: Mm-hmm (acknowledging).
>
> GTA3: And I couldn't really understand their background, so I couldn't understand where they were coming from and where I needed to take them then or where I needed to pick them up from in order to get them where I wanted them to be."

Excerpt 4.3.5

A handful of codes fell into two categories. For example, Excerpt 4.3.6 below was coded as "Valuing knowledge of students to inform instruction" and placed into both the Didactical Orientation category and the Pedagogical Orientation category because it focuses both on assessing a lesson or outcomes and also on adjusting instruction to meet needs of individual learners.

One code, "Unpacking Content Knowledge" fell into all three categories. As Excerpt 4.3.7 shows, excerpts coded within this category reflect a desire to tailor instruction (didactical orientation) based on a student's individual needs (pedagogical orientation) as determined by prerequisite content knowledge (subject matter orientation).

> "Um, so, um, I-I think mainly my teaching, uh, techniques or outlook on it came from more hands-on experience, whether it was my undergrad being in, uh, a classroom and seeing what happens and seeing what's good and bad, um, to teach on my own or tutoring on my own, and seeing like what the-the student understands and what they-they don't. Um, I don't ... I feel like it's kind of hard in a classroom to say like, 'This is good, this is bad' Depending on the student is really how you tell what's gonna help them, what's not."

Excerpt 4.3.6

> "What I struggle with is going back and making sure the foundation material is sound enough to build upon. Sometimes having a better understanding of the background knowledge provides you with more information for the problem at hand."

Excerpt 4.3.7

The full list of codes included in each of the three Beijaard Orientation categories can be found in Appendix G. The relational plot of the codes associated with Beijaard's framework is shown in Figure 4.3.2. It should be noted that the relative distance and size of the vertices is for convenience and implies no mathematical relationship or significance.

The relational plot allows us to gain a sense of the where the participants fall in Beijaard's framework, and summary statistics from RQDA allow us to put some numbers to that picture. Table 4.3.2 indicates the number of distinct codes associated with each category, the number of distinct excerpts (codings) assigned codes within that category, the number of files containing excerpts coded within the category, and the average length of the excerpts within the category. This analysis bears out our expectation that the GTAs cluster largely along the Didactical/Subject-Matter edge of Beijaard's triangle. The more interesting question of shifts over time are addressed in each of the individual case analyses in Chapter 5.

Category	**Codes**	**Codings**	**Sources**
Didactical Orientation	35	231	35
Subject Matter Orientation	30	158	35
Pedagogical Orientation	17	105	20

Table 4.3.2: Summary Statistics for Code Categories Using Beijaard et al.'s Framework. 'Codes' is the number of distinct codes assigned to the category. 'Codings' is the number of excerpts assigned to those codes. 'Sources' is how many data sources contained excerpts using a code from that category.

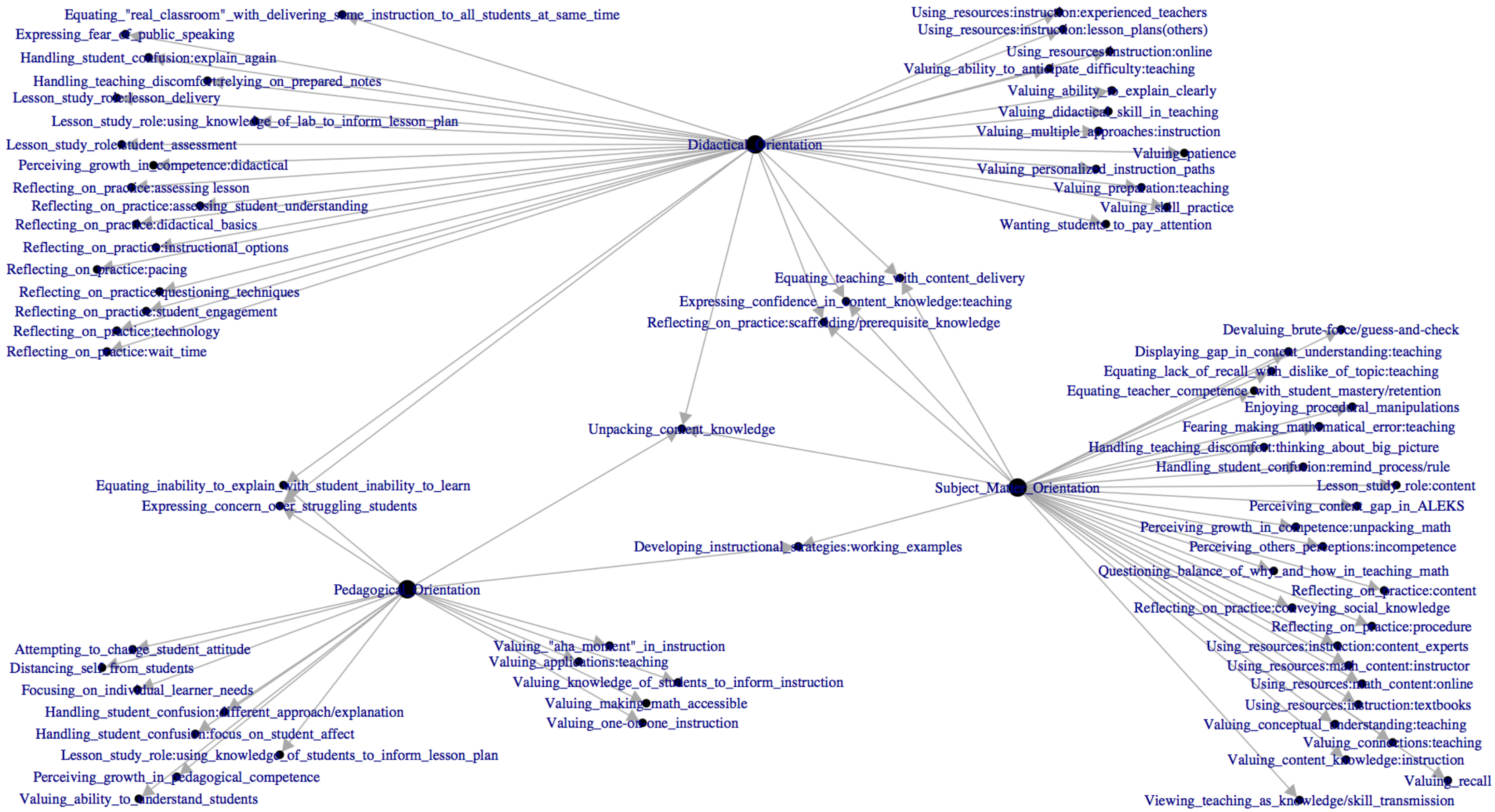


Figure 4.3.2: Interrelation Plot Using Categories Adapted from Beijaard et al.'s Framework for teacher identity. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the three code categories is displayed with a larger vertex size than the codes associated with that category.

4.4 Analysis Using Van Zoest & Bohl's Framework

For easy reference, we include again Van Zoest & Bohl's framework for teacher identity development in Figure 4.4.1 and remind readers that it represents a continuum of social interaction across three domains: content, pedagogy, and professional participation. The continuum of social interaction is broken into two rough chunks: 'Self-in-Mind' and 'Self-in-Community'.

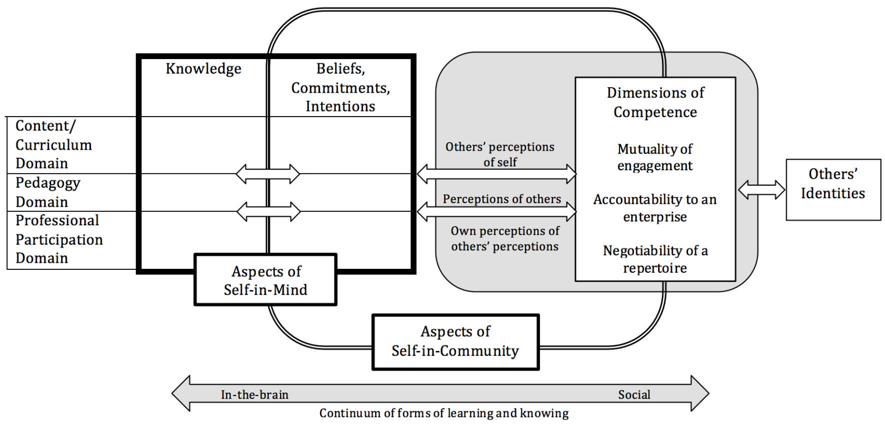


Figure 4.4.1: Van Zoest and Bohl's Dyanamic Model of Secondary Mathematics Teacher Identity Development

Based on this framework, we had originally anticipated six code categories: content, pedagogy, and participation domains within each of self-in-mind and self-in-community. However, while it proved easy to classify codes according to focus within the self-in-mind domains, the groupings associated with self-in-community didn't fall so easily into content/pedagogy/participation domains. Instead, we found that the codes were more appropriately grouped by nature of the interaction (perception versus action) and thus we used the Perception and Competence clusterings within the Self-in-Community portion to the framework to form two domains rather than three. The codes from our qualitative data were thus ultimately grouped into five categories adapted from Van Zoest & Bohl's framework, as described in Table 4.4.1. See Figure 4.4.2 for an interrelationship plot between the code categories.

Category	Criteria	Sample Excerpt
Content Domain (Self-in-Mind)	Pertaining to beliefs about the nature of mathematics and knowledge of the mathematics that is to be taught, including how that content relates to other mathematical content	"And then be able to translate that into an inequality, because, like, in an, in an equality, you have to be able to know that if you do something to one side you have to do something to the other, and be able to work there. And then, once you put the inequality there, we also have to know what the inequality means. Um, that symbol that looks like an alligator or whatever ... -like a sideways V, what does it mean? ... Well, less than or greater than, so it means that. Like, if x is less than 5, then any values that are smaller than 5 will make that true, whether it's a -5, so anything that's smaller than 5, or like if it's $x + 3 < 5$, well, then, any number that will make that true is our answer, so x has to be less than 2."
Pedagogy Domain (Self-in-Mind)	Pertaining to instructional methods, classroom leadership, and ensuring broad development of student understanding	"I'm always trying to understand how students think and the misconceptions and common mistakes they have. By reading the case studies and watching/teaching/tutoring/grading for [Precalculus], I have learned more misconceptions and mistakes."
Participation Domain (Self-in-Mind)	Pertaining to awareness of and interactions with broader professional teaching communities (not just how content connects to other courses)	"[T]he semester as a whole was very humbling. There were many times in [Precalculus] that a question was asked, and I didn't have the answer. It allowed me to find help and gain a deeper understanding of specific topics. I am much more adept at asking for help or researching solutions now, instead of just trying to plow my way through and maybe not explain something well, or worse, explain it wrong."
Perception Domain (Self-in-Community)	Addressing perceptions of others, perceptions of self, and others' perceptions of self with regard to content, pedagogy, and practice; 'others' includes students, peers, and supervisors	"A small seminar thing to get you ready for teaching, and you have to present a mini lecture, and then answer problems on it, and one of the professors asked a question and I just kind of blanked. I didn't know how to convey it. It was one of things like, 'Uh-' In a classroom setting I'd be like, 'I don't know, like let me know, and I'll get back to you the next class.' Um, so that was kind of like frustrating just like at the moment not having a very good answer to it."
Competence Domain (Self-in-Community)	Working within teaching community of practice to develop competence, negotiate roles in teaching, and reflect on outcomes of teaching practice with others	"When graphing the continuous piecewise defined functions, I wasn't quite sure how to explain joining the endpoints. I tried to make it clear that the function was continuous, but I wasn't sure how to get this across and at the same time make clear that each endpoint came from only one of the pieces of the function. Some group reflection on how to deliver this idea would be helpful."

Table 4.4.1: Qualitative Data Code Categories from Van Zoest & Bohl's Framework for teacher identity development, together with criteria for inclusion in a category and samples for each adapted code category.

It is worth noting that where Beijaard et al. separate Didactical and Pedagogical Orientation, Van Zoest & Bohl include both instructional design and support of individual learning needs within the Pedagogy Domain. Thus Excerpt 4.3.1, which was placed in the Didactical Orientation within Beijaard's framework, would fall instead in the Pedagogy Domain within Van Zoest & Bohl's framework.

Because Van Zoest & Bohl consider a social interaction dimension within their framework, we see more crossover between domains, particularly the Pedagogy Domain (Self-in-Mind) and the Perception Domain (Self-in-Community). Excerpt 4.4.1 in all its brevity, provides an insight into why we might see such an overlap. This excerpt was coded as "Valuing ability to help others: math" and that code was deemed part of the Pedagogy Domain since its codings include placing value on helping individual learners gain mathematical understanding or skill. That same code was also deemed part of the Perception Domain because its codings include reference to self-perception or affect associated with the social interaction.

> "And then teaching, er, um, tutoring in college, at college level it was just always nice to see like, when a student finally got it. That was just a really good feeling for me."

Excerpt 4.4.1

As was the case in Beijaard's framework, not all of the codes associated with a category necessarily represent a positive association. For example, within the Perception Domain (Self-in-Community) we see codes such as "Expressing fear of public speaking," "Fearing being seen as incompetent," "Expressing frustration:teaching" and "Distancing self from students" among others. Indeed, the affective components of frustration, fear, stress, confidence, and enjoyment that do not have a place in Beijaard's triangle form a key facet of Van Zoest & Bohl's model.

Sorting affective codes into "self-in-mind" versus "self-in-community" was tricky, as the spectrum from individual to social is a continuum rather than a collection of discrete steps. The deciding factors were generally the level of involvement of others and the degree to which the excerpt aligned with content, pedagogy, or participation. For example, Excerpt 4.4.2 was coded as "Expressing frustration: teaching" and placed within the "Perception Domain (Self-in-Community)" category because it focused on interactions with students and perceptions of frustration and competence related to the interaction.

> “I think ... I mean when they don’t really even like listen to you at all. It’s like, I don’t know. That’s, that’s frustrating I guess. Um, there have been times when I wasn’t exactly sure. Like I knew ... I understood the material, like the math. But I wasn’t sure exactly what they needed to get out of it, I guess.”

Excerpt 4.4.2

Excerpt 4.4.3, on the other hand, was coded as “Handling student confusion: assume teacher fault” and coded within the “Pedagogy Domain (Self-in-Mind)”. Even though it also mentions interaction between teacher and students, it focuses on the internal reaction rather than the external interaction, and it does not have a direct affective component. The decision to place it in the Pedagogy Domain rather than the Content Domain is because it focuses on instructional/didactical aspects rather than on specific mathematical understandings.

> “Usually I assume that if they don’t understand its [sic] because something in my explanation was unclear, so I try to rectify that.”

Excerpt 4.4.3

Some codes, such as “Fearing making mathematical error:teaching” or “Valuing preparation:teaching” landed in both a Self-in-Mind category and a Self-in-Community category. Excerpt 4.4.4 was coded as “Fearing making mathematical error:teaching” and placed within both the Content Domain (Self-in-Mind) and the Perception Domain (Self-in-Community) categories. This coding, like others from the same code, reflects both concern over the mathematical content and also an affective component as it relates to perceptions of self and to others’ perceptions of self in a teaching role. In this case, the ‘others’ are both students and the supervising faculty. Within this particular excerpt, we also see didactical and pedagogical elements, so this excerpt carries other codes as well. The other codings from this code do not necessarily reflect pedagogical or didactical concerns, so the code as a whole was not assigned to the Pedagogy Domain (Self-in-Mind).

> “When I got to the final step and [supervising instructor] pulled me aside, I knew something was wrong, but I wasn’t prepared for ‘you’re doing it wrong’. I thought she was going to say, ‘You are being very unclear, maybe you should stop and rethink this.’ Hearing ‘you’re doing it wrong’ definitely threw me off. I tried to quickly adjust, but of course the students knew that I had done something wrong. I was very concerned that they would then be even more confused on the topic.”

Excerpt 4.4.4

Excerpt 4.4.5 was coded as “Valuing preparation:teaching” and that code was assigned to both the Pedagogy Domain (Self-in-Mind) and the Competence Domain (Self-in-Community). The assignment to the Pedagogy Domain was based on internal realizations about notation as one component of instructional practice, and assignment to the Competence Domain was based on the aspect of negotiating roles and reflecting on outcomes of teaching practice within the community.

> "As far as like personal experience, I had one lecture that I taught, my notation was not consistent with, with how the, what the teacher had been using so that was like a source of, of confusion there which just arose from not, you know, sitting down before and realizing, okay she does this so I need to do this too."

Excerpt 4.4.5

As noted previously, distinguishing between the Perception Domain and the Competence Domain within the Self-in-Community was also challenging at times. The distinction we made was based on whether the focus was on *perception* or on *action*. Thus Excerpt 4.4.2 fell within the Perception Domain since it was focused on the self-perception of frustration and competence. In contrast, Excerpt 4.4.6 was coded as "Feeling pressure/stress:teaching" and that code was placed within the Competence Domain (Self-in-Community). Despite the affective component of pressure or stress, the focus is on taking action to develop competence.

> "I knew going into it that I would probably feel this way and be frustrated with myself and it didn't go well. Um, so I guess I'm trying to just like be patient and realize that they're learning, but I'm also learning how to teach. And so, to,to try and like when it doesn't go well, instead of being frustrated, look at what specifically didn't go well and how to fix that next time. Instead of just focusing on the fact that it didn't go well."

Excerpt 4.4.6

Looking at Figure 4.4.2, we notice that of the five domains adapted from Van Zoest & Bohl's framework, the Participation Domain (Self-in-Mind) is dwarfed by the other four. That observation is reinforced by the summary statistics in Table 4.4.2. This is entirely unsurprising. As noted in Chapter 1, graduate teaching assistants are provided with very little in the way of a teaching community of practice. The concept of a broader community of teaching practice is only at the edges of the awareness for these fledgling teachers. This gap, however, does highlight a potentially valuable field to develop to enhance a sense of teacher identity for GTAs, particularly since most of the codings from this domain are similar to the sample provided in Table 4.4.1.

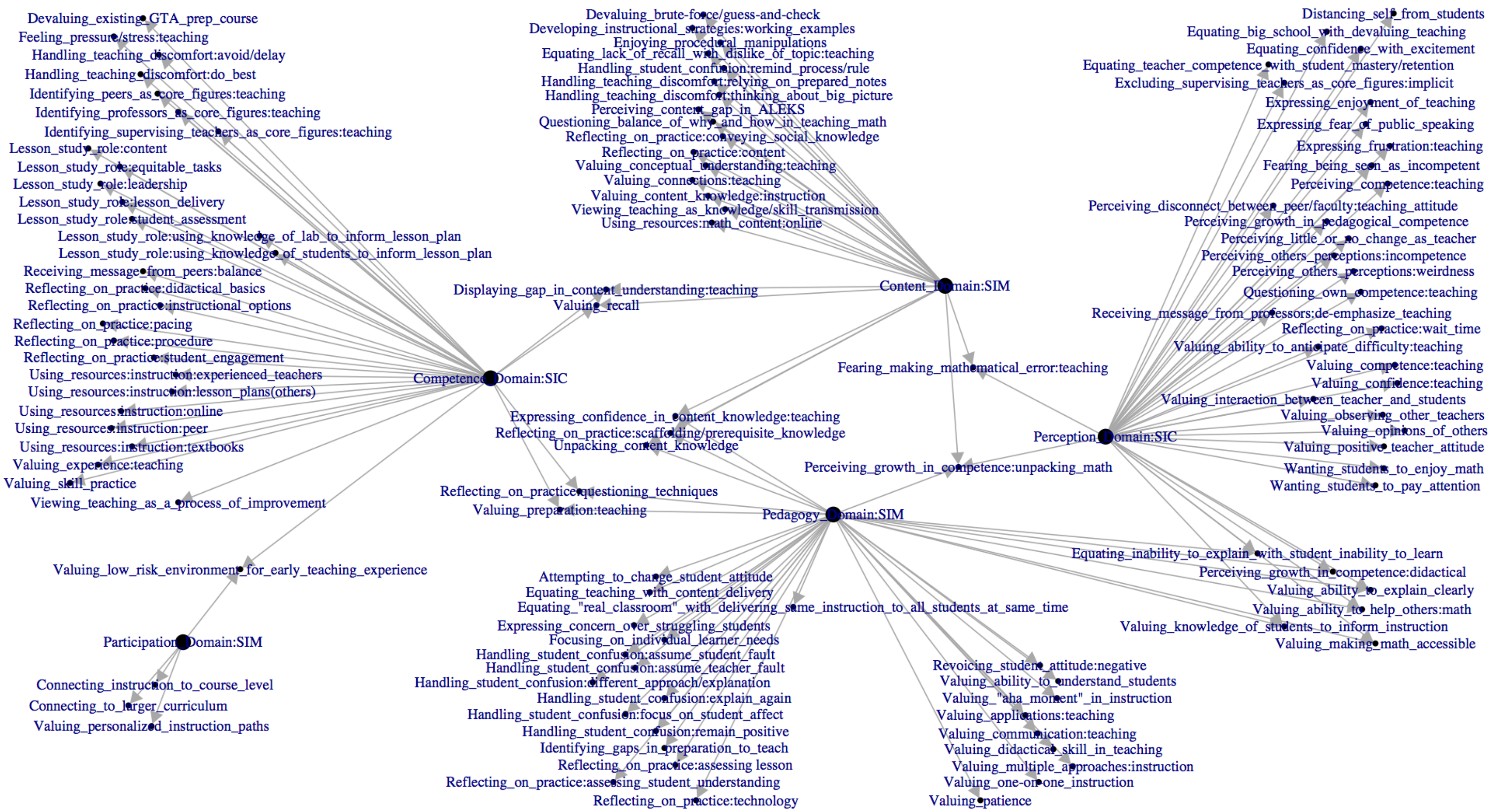


Figure 4.4.2: Interrelation Plot Using Categories Adapted from Van Zoest & Bohl's Framework for teacher identity development. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the five adapted code categories is displayed with a larger vertex size than the codes associated with that category.

Category	Codes	Codings	Sources
Content Domain (Self-in-Mind)	23	135	33
Pedagogy Domain (Self-in-Mind)	36	233	30
Participation Domain (Self-in-Mind)	4	9	6
Perception Domain (Self-in-Community)	35	210	23
Competence Domain (Self-in-Community)	33	217	36

Table 4.4.2: Summary Statistics for Code Categories Using Van Zoest & Bohl's Framework. 'Codes' is the number of distinct codes assigned to the category. 'Codings' is the number of excerpts assigned to those codes. 'Sources' is how many data sources contained excerpts using a code from that category.

4.5 Analysis Using Ronfeldt & Grossman's Framework

As with the previous frameworks, we present again Ronfeldt & Grossman's framework for professional identity development in Figure 4.5.1.

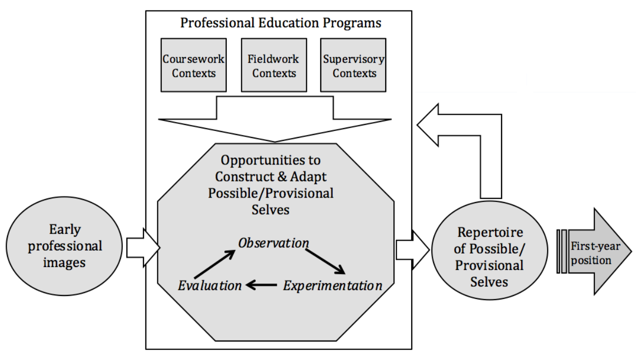


Figure 4.5.1: Ronfeldt and Grossman's model of professional identity development, based on participants in professional education programs in clinical psychology, clergy, and secondary education.

We initially grouped the codes into five categories drawn from this framework: early professional images, coursework contexts, teaching contexts, research/supervisory contexts, and provisional selves. However, the dissonance between a teaching community of practice and a mathematical community of practice became jarringly evident in doing so. The participants' early professional images for 'mathematician' versus 'teacher of mathematics' were so disconnected from each other that they necessitated separate grouping. Moreover, since the data were collected early in the participants' graduate programs, all of their images could reasonably be classified as 'early professional images.' We therefore broke our preconceived "Early Professional Images" category into two separate categories: "Professional Images: Mathematician" and "Professional Images:Teacher". The "Teaching Contexts" category as we initially envisioned it encompassed all of the coded data related to teaching within the graduate school setting. That collection was so large that it gave little useful information regarding experimentation with provisional selves. We ultimately developed and refined six categories adapted from Ronfeldt & Grossman's model, as described in Table 4.5.1. See Figure 4.5.2 for the interrelationship plot between these six categories and Table 4.5.2 for summary statistics on the categories.

Code Categories within Ronfeldt & Grossman's Model of Professional Identity Development		
Category	**Criteria**	**Sample Excerpt**
Professional Images (Teacher)	Participant's views on what it means to be or become a teacher of mathematics, drawn from pre-graduate school experiences; excludes new views on profession gained in graduate school but includes ones that existed and were reinforced	"[K]ind of like what I said with an actual teaching job, where you have a lot more added on ... Even if you have all your planning done, you're a couple years into your teaching position, um, you still have the grading aspect. You still have the answering to students. You still have the ... Depending on what level you're at... A high school ... Uh, the parent interaction if you're in the high school level with checking in on their students or, um, answering to maybe the principal or whoever."
Professional Images (Mathematician)	Participant's views on what it means to be or become a mathematician, drawn from pre-graduate school experiences; excludes new views on profession gained in graduate school but includes ones that existed and were reinforced	"Well I did um, summer research the summer after my ... Was it sophomore year? Um, in college and I pretty much did it cause I was thinking about maybe going to grad school and I knew it would help with that. Um, and I really didn't think I was going to like it that much, but I loved it. And so, I guess that, that like that was my first experience with like working on a problem that you don't have the answer to in the back of the textbook. And that the aspect of like not knowing the answer and bringing like all these different tools and perspectives. Like you get to play and get to work on solving a problem. I really enjoyed it. I think that's kind of what sealed the deal for me probably."
Coursework Feedback	Self-evaluation and feedback related to role as a student in graduate school; excludes narratives without affective or evaluative component; includes new professional images for mathematics	"I mean, I just finished ... Umm, in the Math Department we have to take six prep courses, one of them being an [abstract] algebra course. And, the amount of, there's homework every week. And the tests are really hard, especially for me. And, so, the amount of time that I had to put into that class compared with my other two classes, it's probably double."
Continued on next page		

Table 4.5.1 – continued from previous page		
Category	**Criteria**	**Sample Excerpt**
Teaching Feedback	Self-evaluation and feedback related to role as a teacher in graduate school; excludes narratives without affective or evaluative component; includes new professional images for teaching	"And I was actually surprised by how much I enjoyed them. I kind of always thought I would just be really nervous and kind of want it to be over the whole time but actually it ... I mean, I would say I was nervous before, especially the first time, but once I like got started I actually really liked it kind of more than I thought I would."
Research Feedback	Self-evaluation and feedback related to role as a researcher in graduate school; excludes narratives without affective or evaluative component; includes new professional images for mathematics	"Uh, the full professors [chuckles] they're kind of funny because, you know, once they've gotten tenure, they're like, 'I don't care what everyone thinks.' So, and, they're, some of them are still really good teachers, and, but others, like, even if they're not good teachers, they're not gonna try and be a great teacher because it doesn't really matter [laughs]."
Provisional Selves	Participant's explicitly stated views of how they saw or see themselves and what they see themselves doing in the future; includes motivational values and goals that directly affect choices; excludes general values not explicitly tied to self-image; excludes general statements of values without a personal connection	"To me a mathematician is one who focuses on math, as in they're trying to go further and make math expand, add their own discoveries to it so that, further down the road, someone can look up their paper and be like, 'Oh, look at this. This has been done, now we can go even farther,' and create new math or discover new math, whatever word you want to use. I don't know if I really wanna be that person. But, if you define mathematician as the one who, performs the math [laughs], then, I don't know. I've always loved math, and I've always wanted-, thought it was just fun to do math."

Table 4.5.1: Qualitative Data Code Categories from Ronfeldt & Grossman's Framework for professional identity development, together with criteria for inclusion in a category and samples for each code category.

Ronfeldt & Grossman's model provides valuable insight into the first-year teaching experience as it fits within the larger expectations of graduate school. As noted, we chose not have a single "Early Professional Images" category, but rather to split it into professional images of teachers and professional images of mathematicians as separate categories. It is within the small overlap between these two categories that we see the complexity of this issue. In Excerpt 4.5.1, coded as

"Equating big school with devaluing teaching" we see a distinction being made between research mathematicians and teachers. The phrase 'they didn't have a graduate program or anything' as the cause of a focus on teaching underscores an early professional image of mathematicians as teachers only in the absence of other (presumably more valuable) activities.

> "I would say Clemson seems to emphasize the importance of teaching more than I kind of thought grad school would because I thought most schools would ... I don't know it just ... I was under the impression that bigger schools just didn't consider it important. Like I came from a small liberal arts school and it was very important there but they didn't have a graduate program or anything so the main focus of, you know, all the professors was, was teaching."

Excerpt 4.5.1

On the other hand, where the two professional images categories intersect with the "Provisional Selves" category, we see positive statements about future selves that include both teaching and mathematics components, such as "Viewing teaching as part of being an academic mathematician" and "Projecting ahead: planning academic career." Those codes include codings both of early plans to become a mathematician as a means of teaching (see Excerpt 4.5.2) and of teaching as a means of being a mathematician (see Excerpt 4.5.3).

> "And so I guess I kind of just knew right off the bat that teaching was something a lot of people did and so that's when the idea kind of first entered my mind that a lot of people who like go to grad school in math end up teaching in academia."

Excerpt 4.5.2

> Int: Can you describe for us what you foresee as maybe a typical week in your professional life after you graduate?
>
> GTA3:Oh gosh. Ha ha. I still don't know really if I want to go into academia or go into industry. Um, so I guess it really depends on which of those two but I guess industry I just envision like, 9 to 5, sitting there, solving ... Either doing stats or solving, um, like LP's and then academia I guess kind of similar to what grad school is like, except teaching instead of sitting, being in the class. Teaching, you know, instead and doing research. So, I don't know. I don't really have a very, like, concrete vision."

Excerpt 4.5.3

In neither case, however, do we see a goal of being an academic mathematician fully integrated with a strong teacher identity. Indeed, Excerpt 4.5.3 is the only coding from all of the data that provides any sense of teacher identity integrated into professional mathematician identity, and it is quite fragile in both scope and intent.

We gain some insight into the distinction by looking at the codes associated solely with Professional Images (Mathematicians) and seeing that they largely contain descriptions of skill sets, such as those in Excerpt 4.5.4, or statements of inherent superiority such as those in Excerpt 4.5.5 and 4.5.6.

> "I think obviously, being logical and being able to express yourself coherently and logically. Um, and thinking independently and being able to think about a problem um, in different ways and bring as many different tools as you can to the problem and different approaches to try to solve it. Um, also knowing like a, a context to put the problem in and knowing if there's similar problems that have been looked at. Maybe you can sort of take the same approach."

Excerpt 4.5.4

> Int 1: Um, so when's the first time you thought you might like to be a mathematician?
>
> GTA2: Probably, I don't know, necessarily thought I wanted to be a mathematician but the first time I really started to notice that I enjoyed math and was pretty good at it was in sixth grade. I, like the first time they actually put me in any advanced classes or anything was strictly for math.
>
> Int 2: Mm-hmm (affirmative).
>
> GTA2: So. That was the first time I thought this might be something worth pursuing.
>
> Int 2: Mm-hmm (affirmative). Because you were enjoying it?
>
> GTA2: Yeah. I was enjoying it and then also it was, it was like, it was easy.
>
> Int 2: Mm-hmm (affirmative).
>
> GTA2: Which was strange because you always hear horror stories about math.
>
> Int 1: Mm-hmm (affirmative).
>
> GTA2: And so I thought, if this is easy there is, it's not by just some fluke or something.
>
> Int 1: Mm-hmm (affirmative).
>
> GTA2: I might have like, at least a little, natural gift to it.

Excerpt 4.5.5

> "Um, I just thought I could, like not to sound mean or anything, but that I could do better [laughing] ... [t]han accounting, just I knew that I could do higher math and stuff so, I should probably do that."

Excerpt 4.5.6

Where the Professional Images categories overlap the Provisional Selves category, we see evidence of which aspects of the envisioned career either draw the participant to that area or distance them from it. The codes at the intersection of Provisional Selves and Professional Images of Teachers involve enjoyment, a desire to transmit positive attitudes towards mathematics, and a desire to help others develop mathematical skill and mastery. They are largely based on deriving satisfaction from helping others. Those at the intersection of Provisional Selves and Professional Images of Mathematicians, on the other hand, involve creating knowledge, solving problems, and making connections. They are largely based on competence. That, coupled with Provisional Selves aspects such as valuing opinions of others and valuing respect, sets the stage for a potentially difficult transition to graduate school.

Indeed, the intersection between Provisional Selves and Course Feedback is almost exclusively negative: disliking theory, feeling frustrated, seeing math as difficult and work-intensive. There is no overlap solely between Course Feedback and Professional Images of Mathematicians, so no aspects of those professional images are being reinforced by the coursework in the first year of graduate school, and research experiences are reinforcing only the professional image of valuing open-ended problems. It is thus unsurprising, although disheartening, that at the intersection of Professional Images of Mathematicians, Course Feedback, and Provisional Selves, we find only one code: "Questioning own competence: mathematical." Similarly, at the intersection of Research Feedback and Provisional Selves, we also find only one code: "Disliking research." The first year experiences of these participants reinforced the goals they strove for in becoming more central to the mathematical community of practice, but at the same time sent the message that they were not capable of reaching those goals. Excerpt 4.5.7, as one of 14 codings in this category, gives an idea of the impact of those experiences.

> "But [laughs] you know, once you get into the higher stuff, it's like, 'This is really hard.' I don't know if I could ... Not necessarily that I wouldn't enjoy being a mathematician. But I don't know if, personally, I would be capable."

Excerpt 4.5.7

In contrast, the teaching feedback received by the participants reinforced their early professional images of teaching in several aspects, including valuing content knowledge. We also see several new ideas introduced into the professional image of teaching for the GTA's, under the category of Teaching Feedback. Many of these pertain to the didactical aspects of teaching as Beijaard defines them, and we do see issues of competence and stress arising at the intersection with Provisional Selves. We also see many new value statements at the intersection of Teaching Feedback and Provisional Selves: valuing experience, observing other teachers, patience, preparation, and multiple approaches among others. Clearly the participants are refining their view of what it means to be a teacher in a far different way than they are refining their view of what it means to be a mathematician.

In contrast to the effects of course and research feedback, at the intersection of Provisional Selves with both Professional Images of Teaching and Teaching Feedback, we find two value codes: valuing the 'aha moment' in instruction, and valuing interaction between teacher and students. These represent early images that have been reinforced by graduate teaching experience, and which are explicitly included within views the participants have of themselves and their futures, as demonstrated in Excerpt 4.5.8.

> "And then in the fall, I was in [supervising teacher's precalculus] lab, and so I got a more of an interaction with students, and, um, kind of a personal tutoring for that, and then, also, interacting with other TA's, cause there's multiple, um, TA's in her cl ... her course ... Um, and I enjoy that more. Um, and then this past fall, I had my own course that I was teaching, and, uh, it definitely required more work. I kinda ... I kinda had a large course or a large class for my first one. I had 44 students after the drop period."

Excerpt 4.5.8

Ronfeldt & Grossman's model gives us the most nuanced view of the complex and conflicting demands of graduate school, and perhaps the best insight into how graduate students balance those demands in the first year. At the aggregate level, some of those connections are actually obscured because it is within this framework that we see the biggest distinctions between how the four participants experienced their first year. Yet it is also this framework that yields the strongest meaningful common themes among the four cases. At the aggregate level, we see from the summary statistics in Table 4.5.2 that the participants came in with quantitatively similar professional images of mathematicians and teacher. By the nature of the data we collected, it is natural that the quantity of teaching feedback far outweighed the quantity of research and course feedback. However, that gap is worth exploring, as it may go farther towards addressing the stark differences in qualitative outcomes for teaching identity versus mathematician identity during the first year. While we note that teaching identity seemed to mature considerably during this year, we would be well served to understand better how to allow that maturation within the context of a mathematician identity rather than apparently at its expense.

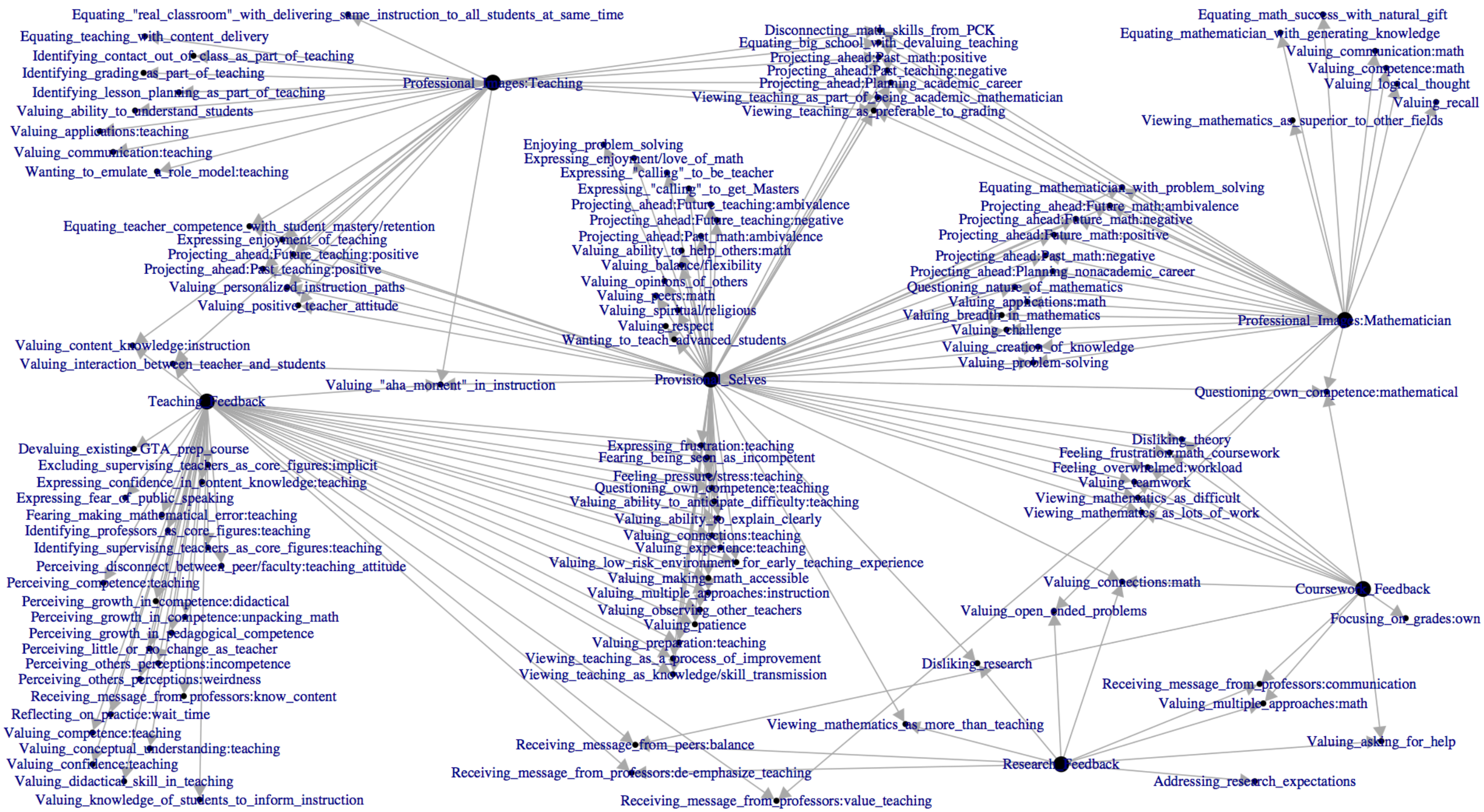


Figure 4.5.2: Interrelation Plot Using Categories Adapted from Ronfeldt & Grossman's Framework for professional identity development. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the adapted code categories is displayed with a larger vertex size than the codes associated with that category.

Category	Codes	Codings	Sources
Professional Images (Mathematician)	28	106	12
Professional Images (Teacher)	24	143	17
Coursework Feedback	13	60	9
Teaching Feedback	44	293	30
Research Feedback	10	20	7
Provisional Selves	65	403	27

Table 4.5.2: Summary Statistics for Code Categories Using Ronfeldt & Grossman's Framework. 'Codes' is the number of distinct codes assigned to the category. 'Codings' is the number of excerpts assigned to those codes. 'Sources' is how many data sources contained excerpts using a code from that category.

4.6 Alignment of the Three Frameworks

In some sense, the three frameworks we are considering can be stacked by granularity. Beijaard et al. decompose a static sense of teaching into three components taken largely in isolation. Van Zoest & Bohl consider teaching as a process of identity development situated within a community of practice. To that extent, their model subsumes Beijaard's, but loses detail in the process. Ronfeldt & Grossman's model deals with professional identity development, also situated within a community of practice, but their model is more general applicable than just to teaching identity so, in a sense, subsumes Van Zoest & Bohl's, but again at the expense of finer detail specific to teaching. However, if the models are valid, there should be points of alignment between the layers from general to specific. Where that alignment fails, we must probe more deeply and question either the validity of the models or the cause of the slippage. It is at those gaps that we find deeper understanding of the models themselves, possible issues with our analysis and interpretation of the data, and open questions for further exploration.

We start by considering Beijaard et al. versus Van Zoest & Bohl. We might well expect strong alignment between the Content Domain (Self-in-Mind) and the Subject-Matter Expert Orientation categories, and we note that where Beijaard et al. separate Didactical and Pedagogical Orientation, Van Zoest & Bohl include both instructional design and support of individual learning needs within the Pedagogy Domain. Thus for example, Excerpt 4.3.1, which was placed in the Didactical Orientation within Beijaard's framework, would fall instead in the Pedagogy Domain within Van Zoest & Bohl's framework. On the other hand, Excerpt 4.6.1, which was coded as both "Valuing making math accessible" and as "Handling student confusion: different approach/explanation", was placed in Pedagogical Orientation under Beijaard's framework and also in the Pedagogy Domain (Self-in-Mind) for Van Zoest & Bohl's framework.

> I had a bunch of different athletes in different sports, but I was always like ... I-I had kind of a personal relationship where I knew what they liked besides sports, and I could say, "All right, let's just change the problem. Let's talk about this." And they would get it [snaps fingers] pretty quickly. And that's always what I've thought. Like, if you can relate it to the person, whatever their expertise is, um, sports, or in the case of this one student this previous semester, reading, they have a better handle of it, and they can understand kind of where you're trying to go with it. Like for the one student, um, really engaged in English, miles per hour isn't necessarily as good a tool as like pages per minute, but it's the exact same concept. Um, so, presenting a problem to her like that, and saying it's the same thing as miles per hour, dollars per minute or dollars per year. Whatever you wanna do. Um, in terms of derivatives for the course I just taught, um, just getting them all to something they understand, they're comfortable with, um, I think that's kind of the biggest thing. Kind of going to their realm and saying, "You just crushed this problem. I really didn't help you that much." It's the exact same thing.

Excerpt 4.6.1

However, it is not as simple as all of the Didactical and Pedagogical Orientation codes from Beijaard's model falling neatly into Van Zoest & Bohl's Pedagogy Domain, nor all of the Subject-Matter Orientation codes falling into the Content Domain. Since Beijaard et al.'s model does not include a dynamic development or social component, the codes from each of the Orientation categories may well be split between Self-in-Mind domains and Self-in-Community domains. The category map with all eight categories becomes nearly unreadable. Instead, we connect the Beijaard orientation categories to Van Zoest & Bohl's Self-in-Mind domains in Figure 4.6.1 and separately to Van Zoest & Bohl's Self-in-Community domains in Figure 4.6.2.

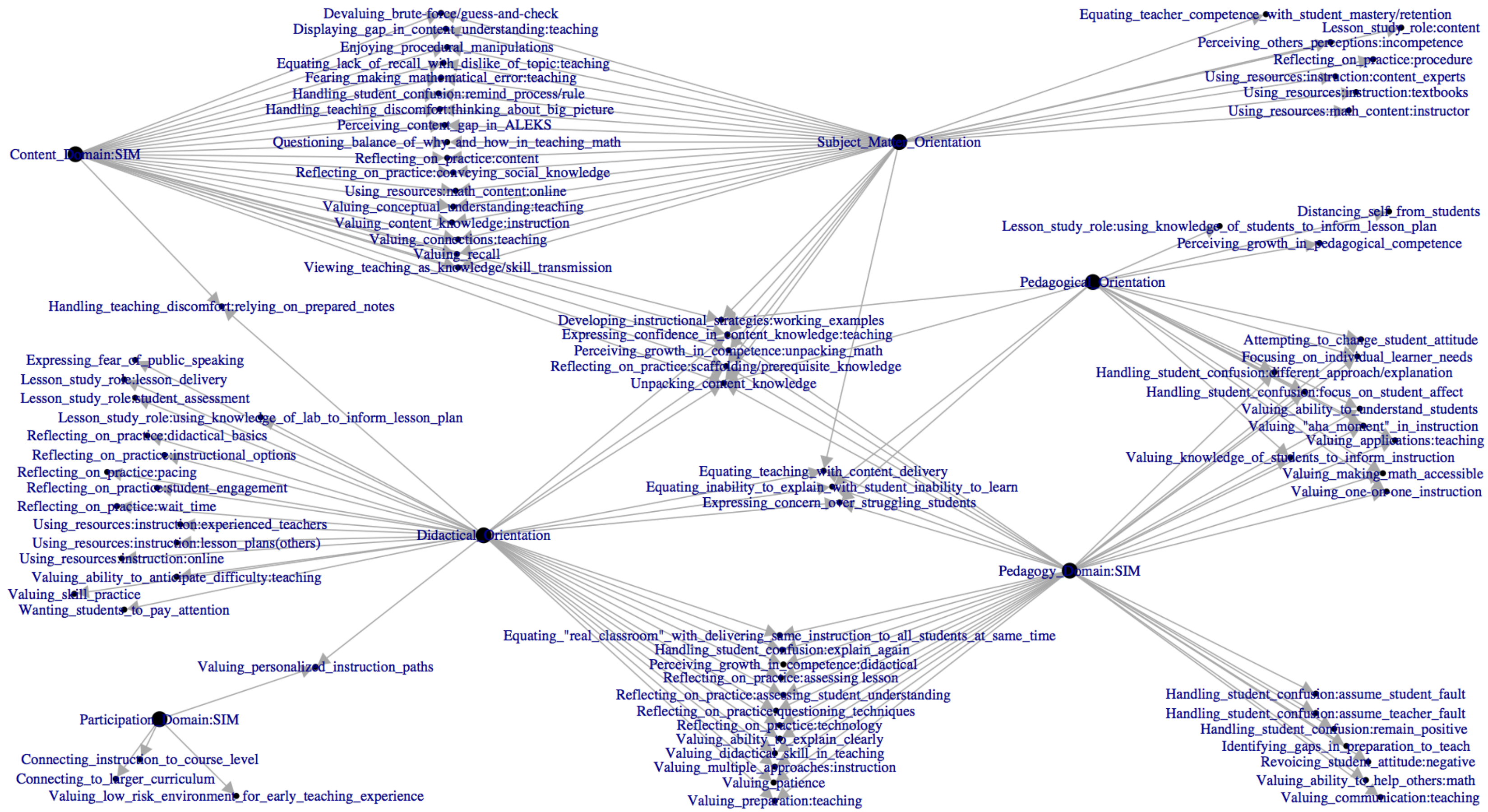


Figure 4.6.1: Alignment Plot: Beijaard et al. and Van Zoest & Bohl (Self-in-Mind). Codes associated Beijaard's Orientation categories and with the Content, Pedagogy, and Participation Domains (Self-in-Mind) from Van Zoest & Bohl's framework for teacher identity development. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the code categories is displayed with a larger vertex size than the code categories associated with that category.

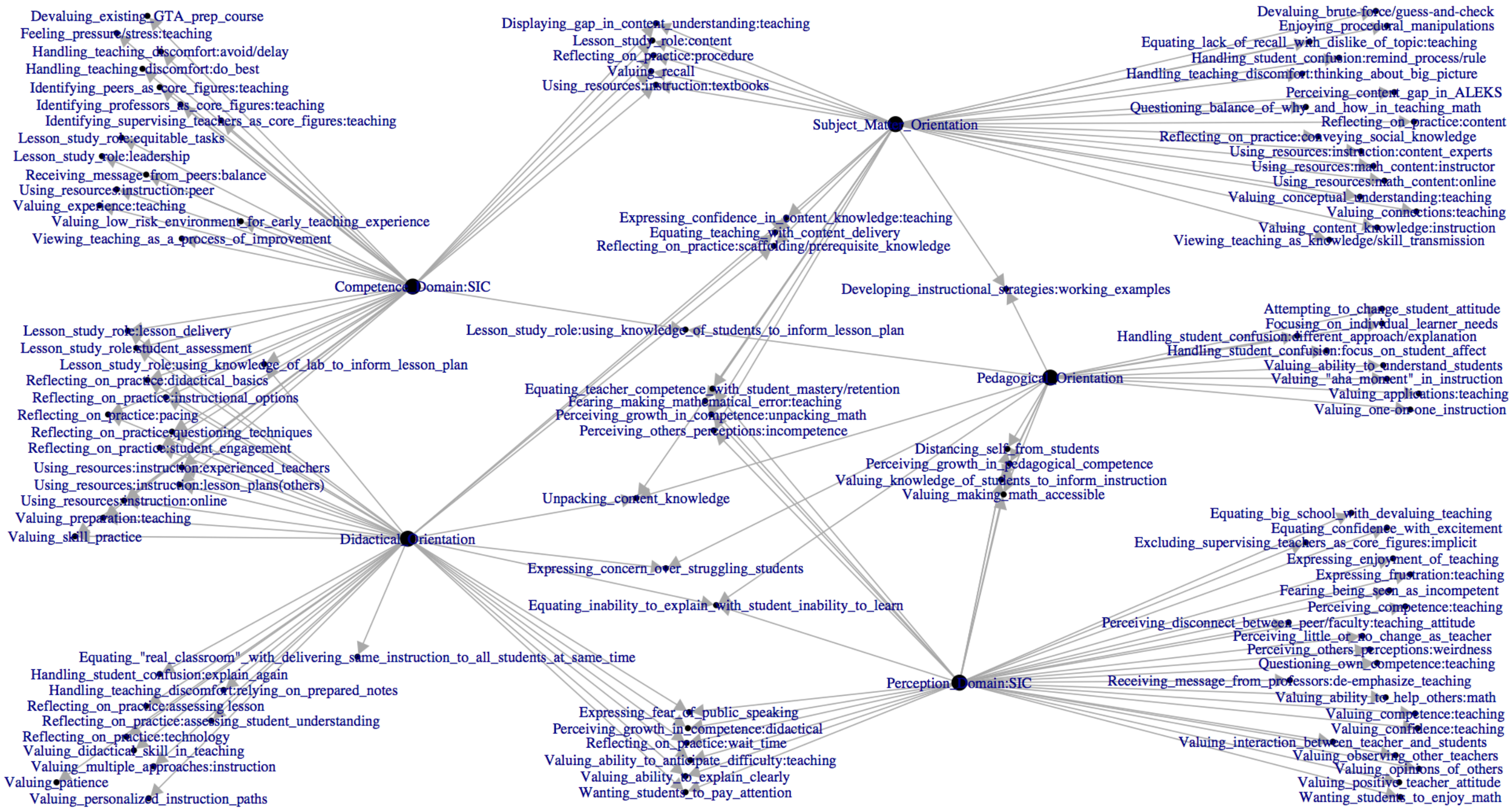


Figure 4.6.2: Alignment Plot: Beijaard et al. and Van Zoest & Bohl (Self-in-Community). Codes associated Beijaard's Orientation categories and with the Perception and Competence Domains (Self-in-Community) from Van Zoest & Bohl's framework for teacher identity development. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the code categories is displayed with a larger vertex size than the code categories associated with that category.

The codes that concern us in this framework alignment analysis are those that are present in one of Beijaard et al.'s orientation categories, but not in any of Van Zoest & Bohl's domains. We find that there are none, although we do indeed see a split between alignment with Self-in-Mind domains and Self-in-Community domains. Note that there are codes that appear in Van Zoest & Bohl's framework but not in Beijaard et al.'s. Those include most of the codes from the Participation Domain (Self-in-Mind) and many of the codes from both Self-in-Community domains. All of them reflect codes associated with the social aspects of identity development within a community of practice, and are appropriately excluded from Beijaard et al.'s categories.

We next turn our attention to the alignment between Van Zoest & Bohl's model and our adaptation of Ronfeldt & Grossman's model. Since both models are based on situated practice, we would expect alignment between the Self-in-Community aspects of Van Zoest & Bohl's model and the teaching components of Ronfeldt & Grossman's model. Since Ronfeldt & Grossman do not include a finer degree of detail for the professional images as they might align with the Self-in-Mind aspects of Van Zoest & Bohl's model, we do not expect to see alignment there. In Figure 4.6.3 we provide a category plot for the codes associated with the Perception Domain (Self-in-Community) and Competence Domain (Self-in-Community) from Van Zoest & Bohl, and the Professional Images (Teachers), Teaching Feedback, and Provisional Selves categories from Ronfeldt & Grossman's model.

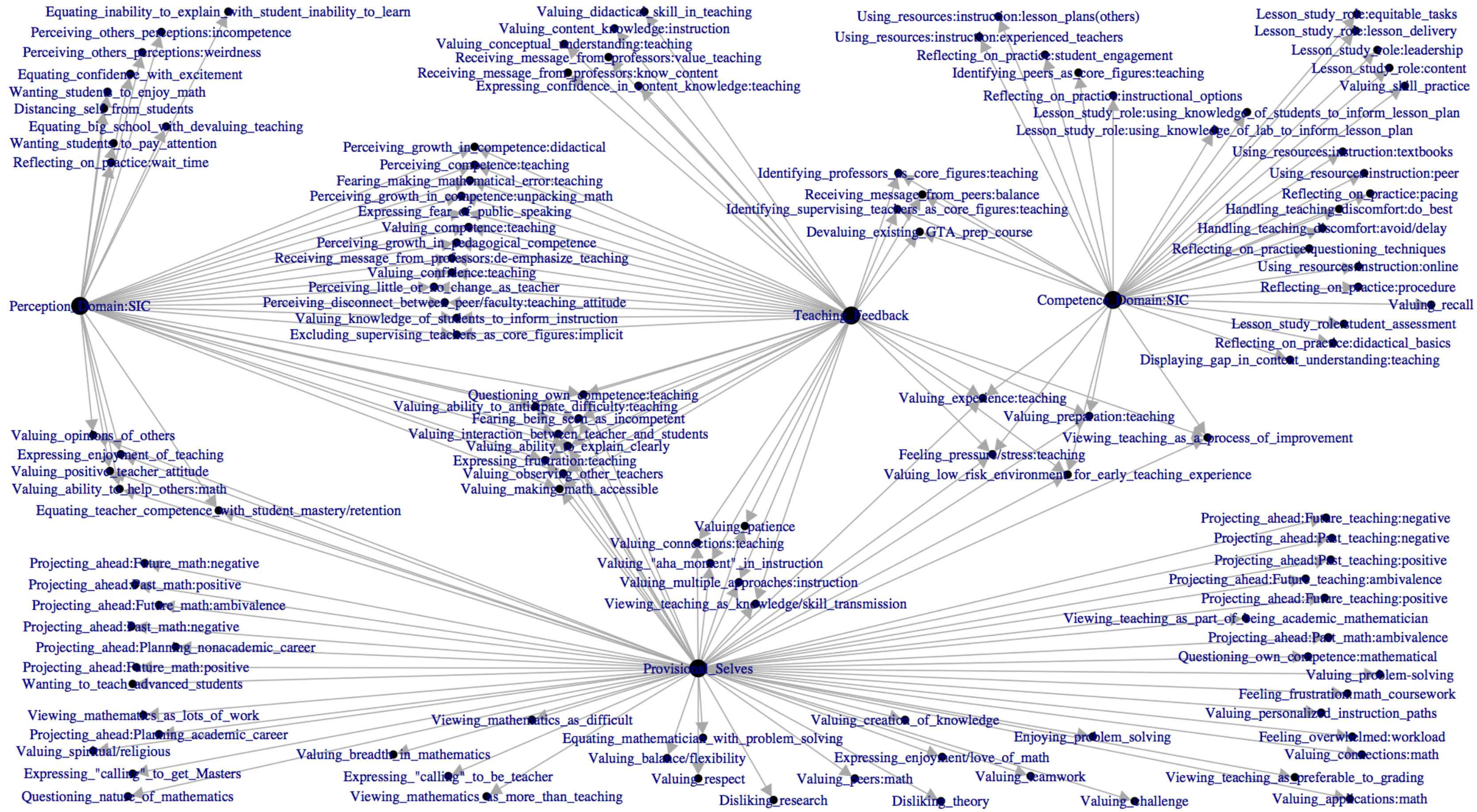


Figure 4.6.3: Alignment Plot: Van Zoest & Bohl and Ronfeldt & Grossman. Codes associated with the Perception Domain (Self-in-Community) and Competence Domain (Self-in-Community) from Van Zoest & Bohl's framework for teacher identity development and the Teaching Feedback and Provisional Selves categories from Ronfeldt & Grossman's framework for professional identity development. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the code categories is displayed with a larger vertex size than the codes associated with that category.

Here again, we concern ourselves primarily with codes that are anchored in Van Zoest & Bohl's model, but not in Ronfeldt & Grossman's. However, we also remind the reader that we excluded from the Teaching Feedback category of Ronfeldt & Grossman's model those codes that contained neither an affective nor an evaluative component. There were many of those from the lesson study reflections and written reflections. They fit appropriately within Van Zoest & Bohl's model and an argument could be made for including them in Ronfeldt & Grossman's model. However, since they appeared to obscure the provisional self loop within that model, we did exclude them. As a result, many of the codes from the Competence Domain of Van Zoest & Bohl's model are not reflected in Ronfeldt & Grossman's model. We believe this degree of non-alignment is appropriate under the circumstances and that, although these excerpts do reflect teacher identity development, they do not strongly reflect professional identity within the larger graduate school experience in that there is no evidence of impact on provisional selves.

There are also several codes from the Perception Domain (Self-in-Community) of Van Zoest & Bohl's model that are not reflected in Ronfeldt & Grossman's model. Some of these are aligned with Professional Images (Teachers) from images prior to the graduate school experience. Others were excluded from Ronfeldt & Grossman's model because we chose to include only those experiences that occurred during graduate school, whereas we included pre-graduate school experiences in Van Zoest & Bohl's model. This bring to the forefront that fact that we are using Van Zoest & Bohl's model to explore teacher identity development, both specifically as a single profession and generally across the full span of relevant experience. In contrast, we are using Ronfeldt & Grossman's model to explore teacher identity development as only one component of larger professional identity development, but within a temporally limited span. Within those constraints, we believe that there is strong alignment at appropriate connection points between the two models.

Given the very different nature, intent, and scope of Beijaard et al. and Ronfeldt &Grossman's model, we do not find an analysis of the direct alignment between those models to be productive. However, the alignment between the models we do compare lends considerable strength to the validity of the analysis and to the conclusions drawn from that analysis.

Chapter 5

Individual Case Analyses

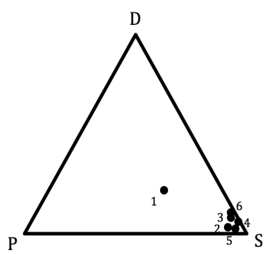
Previously, we have simply referred to GTA1, GTA2, GTA3, and GTA4. For the sake of personalizing each of these participants, we give them pseudonyms for this and subsequent chapters, using the first four letters of the alphabet to correspond to the participant numbers used so far. Thus GTA1 becomes "Anna," GTA2 becomes "Bill," GTA3 becomes "Cora," and GTA4 becomes "Dave."

We begin each section with a restatement of the quantitative survey summary changes from Section 3.1 and the Beijaard identity trajectory drawn from analysis of the case arcs in Section 4.1. We next provide a table of prompts for which the participant's answers changed from Agree to Disagree, or vice versa, between survey administrations. We chose not to include in this chapter the prompts that differed in intensity but not direction, such as Agree versus Strongly Agree. However, the complete tables of prompts for which responses differed can be found in Appendix J.

Within each table, the prompts are clustered into three categories: Images (Math), Images (Teaching), and Beliefs (Teaching). Images (Math) corresponds to items affecting the mean change in mathematician identity, and includes prompts about the images participants have about mathematics professors as well as their mathematical affect and self-efficacy. It includes prompts such 'My mathematics teachers present material in a clear way.' Images (Teaching) parallels the Images (Math) but with a focus on teaching. It includes prompts such as 'Math professors spend very little time thinking about teaching.' Beliefs (Teaching) corresponds to items affecting the mean change in epistemology and includes prompts related to how students best learn mathematics as a proxy for capturing participants' views on the nature of mathematical knowledge. It includes prompts such as 'Students should be told to solve problems the way the teacher has taught them.'

For each case individually, we integrate the quantitative and qualitative data to capture to the extent possible the essence of the participant's first year experiences with respect to teaching in graduate school. Participant-specific category plots for the coded interview and reflection data are embedded within each narrative.

5.1 GTA1 (Anna)



GTA 1

Mean Mathematician Identity Change: -0.255813953

Mean Teacher Identity Change: $+0.060606061$

Mean epistemic Change: -0.08

	✓	**Prompt**	**Pre-**	**Post-**
		I remember most of the things I learn in mathematics.	Agree	Disagree
Images (Teaching)		Math professors spend very little time thinking about teaching.	Agree	Disagree
		Math professors are expected to spend most of their time on research.	Disagree	Agree
		I feel at ease talking about teaching.	Disagree	Agree
		I get bored when people talk about different ways to teach.	Agree	Disagree
		No matter how hard I try, there are some math topics I cannot teach well.	Disagree	Agree
		I enjoy talking to other people about teaching.	Disagree	Agree
		I am good at teaching.	Disagree	Agree
		It scares me to have to teach math.	Disagree	Agree
		I would be happy if I never taught math.	Disagree	Agree
		I think of myself as a teacher.	Disagree	Agree
Beliefs (Teaching)		Students should be allowed to invent ways to solve math problems before the teacher demonstrates how to solve the problems.	Agree	Disagree
		The instructional scope and sequence of math topics should be determined by the formal organization of mathematics.	Disagree	Agree

Continued on next page

	✓	Prompt	Pre-	Post-
Table 5.1.1 – continued from previous page				
		Students should master computational procedures before they are expected to understand how those procedures work.	Disagree	Agree
		Students learn mathematics best by figuring out for themselves the ways to find answers to math problems.	Disagree	Agree
		Students should be told to solve problems the way the teacher has taught them.	Agree	Disagree

Table 5.1.1: Table of Selected Survey Item Results for GTA1 (Anna). Pre- and post-survey prompts for which Anna's responses changed from Agree to Disagree or vice versa. The survey was forced-choice with four options: Strongly Agree, Agree, Disagree, Strongly Disagree. Items that differed by two steps are indicated by a checkmark (✓) in the '✓' column.

Anna started her undergraduate studies as a physics major, having never thought about mathematics as a possible major. When she discovered she didn't like physics as much as she had expected, her advisor suggested that she try math; to her surprise, she loved it. A summer research experience after her sophomore year solidified her decision to attend graduate school.

> "Well I did um, summer research the summer after my ... Was it sophomore year? Um, in college and I pretty much did it cause I was thinking about maybe going to grad school and I knew it would help with that. Um, and I really didn't think I was going to like it that much, but I loved it. And so, I guess that, that like that was my first experience with like working on a problem that you don't have the answer to in the back of the textbook. And that the aspect of like not knowing the answer and bringing like all these different tools and perspectives. Like you get to play and get to work on solving a problem. I really enjoyed it. I think that's kind of what sealed the deal for me probably."

Anna entered graduate school with the intention of obtaining a Ph.D. in mathematics and then pursuing a job in industry. She had no teaching or tutoring experiences prior to graduate school, and had taken no instructional theory or pedagogy courses. On the initial survey, she replied 'Strongly Disagree' to the prompt 'I think of myself as a teacher' and 'Agree' to the prompt 'I think of myself as a mathematician.' She and Bill both participated in a summer bridge program prior to the start of fall classes, intended to remedy academic deficiencies from their undergraduate mathematics studies.

Within the combined course and seminar interactions with undergraduate PSTs, Anna made an initial foray into didactical and pedagogical contributions, then retreated to a purely subject-matter expert role in case discussion and stayed there. Within the lesson study experiences, she viewed her contributions as providing content expertise and knowledge of the structure of the lab so that others could plan an effective lesson.

> "I think my main contribution to the first lesson study cycle was giving the first lesson delivery. Since I haven't had any education classes, or much experience teaching, the process of creating a lesson plan was new to me. I tried to let the other members of the group decide how to structure the lesson into a coherent whole that the students would understand. I did help with coming up with the specific examples to use in the lesson though. I feel like I was able to contribute knowledge of how the labs are structured, and how ALEKS works, so that we were able to tailor our lesson plan to that environment."

Despite this, she showed a movement *towards* a stronger teacher identity and *away* from a stronger mathematician identity based on the pre/post-survey. Indeed, she replied 'Disagree' at the start of the semester and 'Agree' at end of the semester to the survey prompt 'I am good at teaching.' At the same time, Anna is the only one who demonstrated a movement *away* from a constructivist view of mathematics and towards a view that students should copy worked examples rather than exploring to develop their own methods. Nonethess, she wrote on her post-survey:

> "Over the course of the semester I've become more confident about explaing [sic] math. I've also come to believe its [sic] more important to present material in an organic way that allows students to understand where the material is coming from, rather than just memorizing how to do it."

This apparent contradiction between the student's self-perception and the quantitative change in the epistomological scale is likely linked to her focus on mastering didactical skill as one route to effective teaching. See Figure 5.1.1 for a map of her profile within Beijaard's framework, using interview, lesson study, and written reflection sources. Although her case-based identity trajectory is solidly anchored at the subject-matter pole, that trajectory takes into account only the extent to which she is willing to voice an opinion within the combined course community of practice. Within the wider range of data sources, we find a heavy leaning toward the didactical end.

In fact, during the first interview midway through her first semester in the program, Anna was still hesitant about even discussing didactical issues, and her comments tended toward the vague:

> "Um, and so when you're teaching, if you can kind of anticipate where the problem spots are and walk them through that. Um, I just feel like it probably helps them understand better."

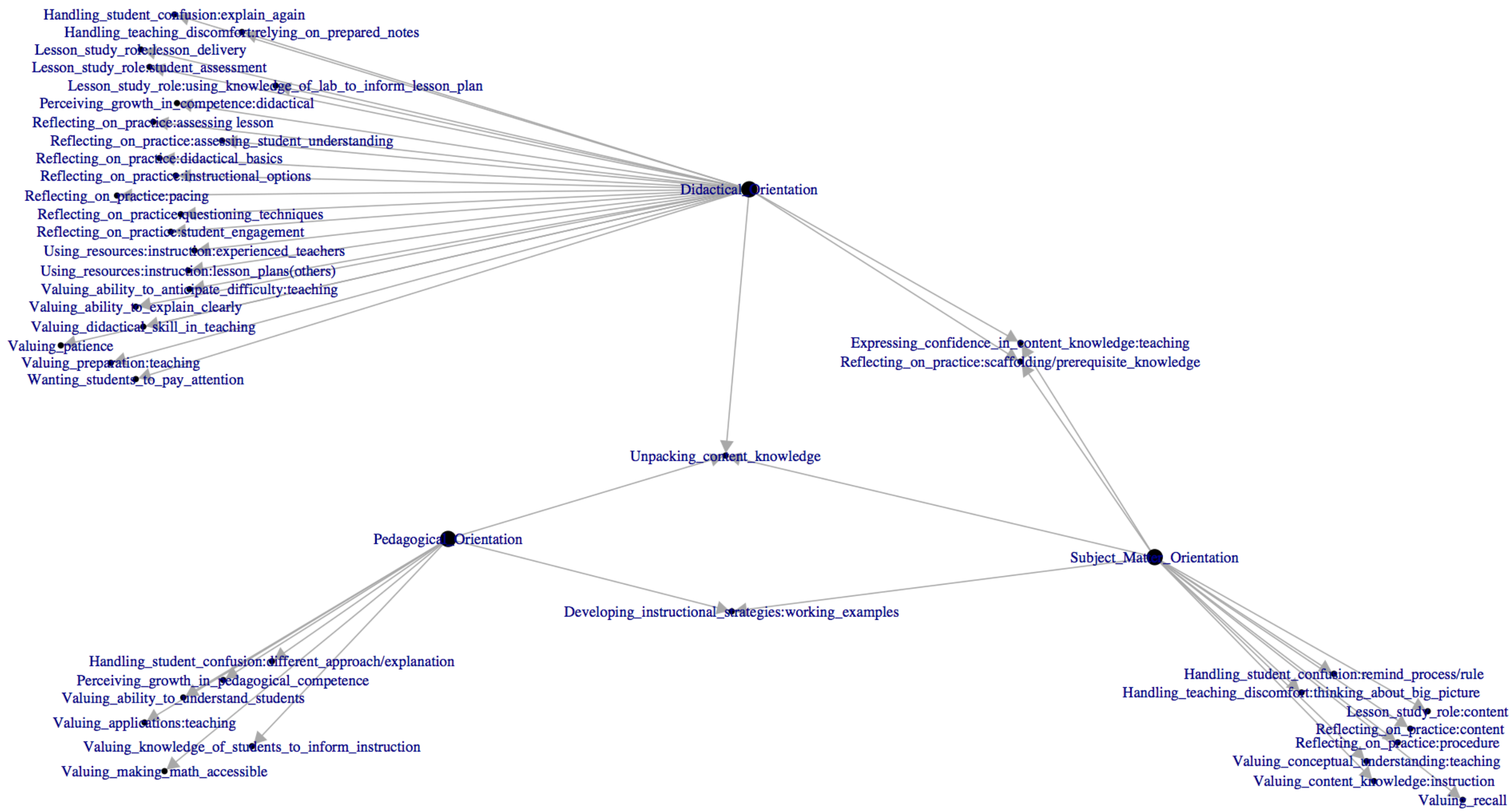


Figure 5.1.1: Anna's profile using codes associated with each of Beijaard et al.'s three orientation categories and incorporating data from both interviews, both lesson studies, and all six written reflections. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the three code categories is displayed with a larger vertex size than the codes associated with that category.

By the end of the first semester, she was more articulate, writing in her final reflection prompt:

> "I learned how important it is to time when to jump in and explain something to a student. When working through problems with students in [precalculus], it became clear that if you jump in to [sic] early to correct a mistake in the way a student is approaching the problem at hand, you sometimes end up confusing them more because they don't understand what they're doing wrong or how it is different from the correct approach. On the other hand, if you wait too long, they get confused and have to completely start over."

By the end of her second semester, her experience working as a teaching assistant in a calculus classroom and participating in a weekly seminar on teaching practice had allowed her to identify specific didactical aspects that she valued.

> "Um, I think, let's see ... Seeing ... Well, first of all in the, the teaching class we all kind of watched each other give our practice lectures and so seeing kind of the same, everyone make the same sorts of mistakes, like erasing too fast or, you know, just clearly being, um, being a little flustered or things like not preparing well ... Things like that, the practical things that you can fix, um, I guess that kind of drove home how important it is and really how easily preventable most of those things are if you're aware of them."

During that first semester, she also began wrestling internally not with whether students should be encouraged to develop their own (mathematically correct) approaches, but rather with how to allow that to occur without permitting errors to go unchecked. Her view of teaching and of her own competence as a teacher are clearly in flux. From the perspective of practical delivery of instruction, Anna both experienced and perceived significant growth in her first year as a mathematics graduate student teacher. At the time of the second interview, she was beginning to perceive her students themselves as a valuable source of information for adapting instruction to meet individual learner needs.:

> "Sometimes when a student doesn't understand what I'm teaching, in the process of working through examples with them, or looking at their work, it becomes clear that what they don't understand is not the topic itself, but some prerequisite material needed for the topic."

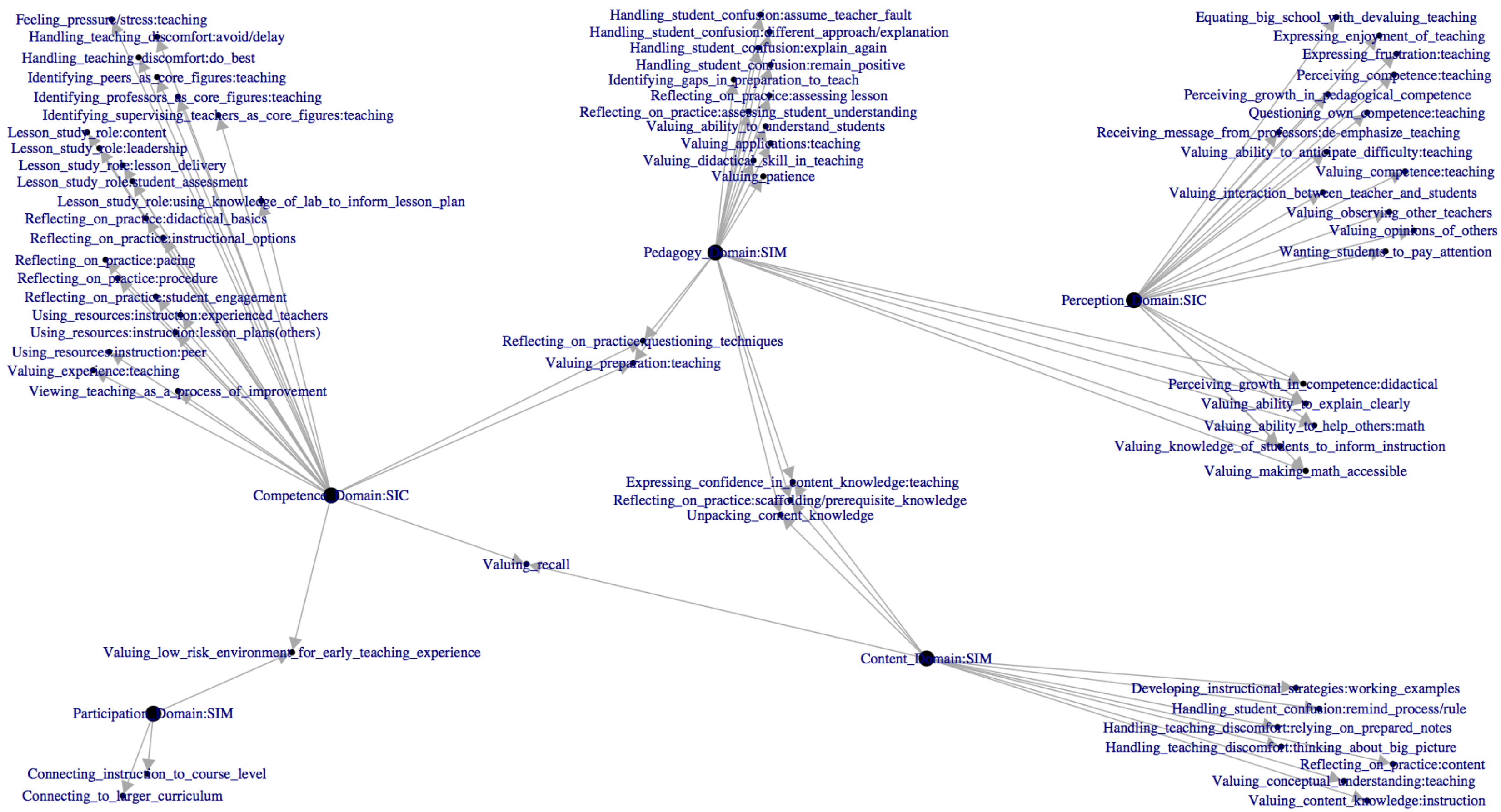


Figure 5.1.2: Anna's profile using codes associated with each of the five categories adapted from Van Zoest & Bohl's framework for teaching identity development, and incorporating data from both interviews, both lesson studies, and all six written reflection prompts. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the five code categories is displayed with a larger vertex size than the codes associated with that category.

Given that significant shifts in identity and teaching practice occur at the boundary between undergraduate studies and first teaching experience for secondary teachers [Beauchamp and Thomas, 2011], and that the "productive friction" produced at those boundaries is an important facet in developing effective instructional practice [Ward et al., 2011], this actually speaks to strong potential for this participant to develop into an effective teacher. That potential appears to be bearing fruit: at the end of the first solo teaching experience, Anna's student performance as measured by overall course grade was on par with that of experienced full-time lecturers, and higher than that of all other GTAs teaching the same course regardless of prior teaching experience (see Figure 3.2.1). We turn to her profile in Van Zoest & Bohl's framework (see Figure 5.1.2) for a clearer view of her growth as a teacher in the first year.

Anna has a preponderance of her codings within Self-in-Community, particularly the Competence Domain, indicating considerable attention given to negotiating roles in teaching and to developing expertise within the teaching community of practice. This is reflected in changes to survey items related to talking to others about teaching (see Table 5.1.1). It also becomes clear that she views teaching as a process of improvement and sees value in working towards that improvement. When asked how she would handle teaching math with which she felt uncomfortable, she replied:

> "I would just do the best that I could, and remind myself that if I never practice teaching topics I'm not fully comfortable with, I'll never get any better at it. After teaching I would take a minute to reflect on what went well, and what needs improvement. Specifically, I would use questions students had or things that confused them to determine which parts of the explanation could be made more clear."

That view of discomfort as an opportunity to improve carries over to her view of mathematics, how she responded to course and research feedback, and her provisional selves from Ronfeldt & Grossman's framework (see Figure 5.1.3.) Although she expressed frustration with her coursework, she also indicated an underlying confidence and persistence.

> "My response when I don't understand what's being taught in a math class varies. I used to get really frustrated if I didn't understand every step. I would get bogged down in understanding all the details and loose [sic] sight of the big picture. As I take harder math classes in my junior and senior years in college, I got used to not understanding every time right away. I learned to follow the big picture in class, and work through the details to deepen my understanding later. It no longer frustrates me when I don't understand, because I know I can figure it out eventually."

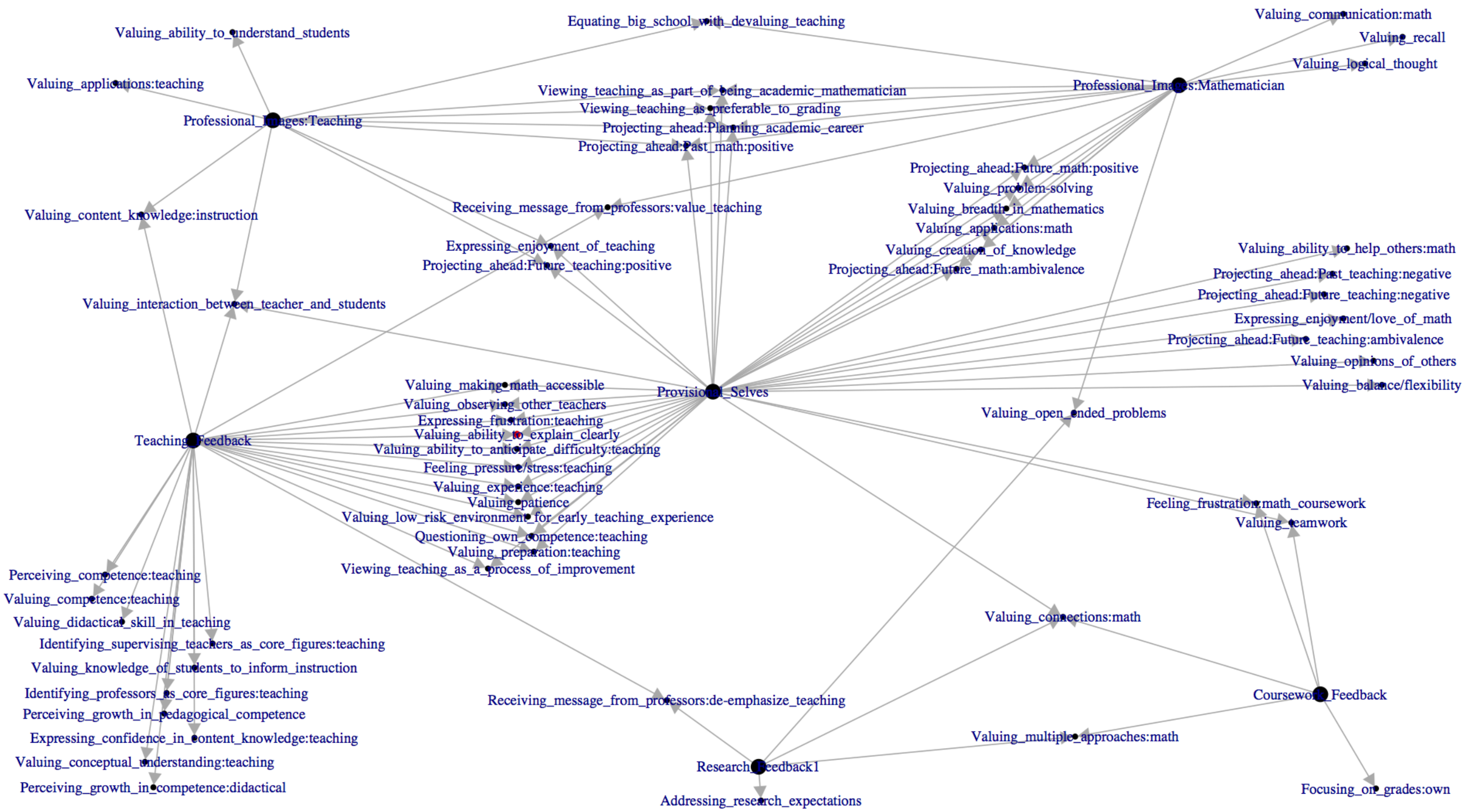


Figure 5.1.3: Anna's profile using codes associated with each of the six categories adapted from Ronfeldt & Grossman's framework for professional identity development, and incorporating data from both interviews, both lesson studies, and all six written reflection prompts. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the six code categories is displayed with a larger vertex size than the codes associated with that category.

Anna started graduate school intending to obtain a Ph.D. and pursue a non-academic career. By the end of the first year, both of those were in flux. Her response to the prompt 'I think of myself as a mathematician' had changed from 'Agree' at the start of the semester to 'Strongly Agree' at the end of the first semester. Over the same time span, her response to 'I think of myself as a teacher' changed from 'Strongly Disagree' to 'Disagree.' She felt both more like a mathematician and also more like a teacher. Although she now saw herself as good at teaching and enjoyed talking to others about teaching, she was also now scared of teaching, thought there were topics she might not be able to teach well no matter how hard she tried, and could see being happy not teaching (see Table 5.1.1.) This state of uncertainty most likely continued through the next semester, as during the second interview she noted,

> "Um, I don't know. I mean, I'm still, I still don't even know if I want to stay for a PhD or just get a Master's and it kind of just depends on the week. If it's an easy week then sure and if it's a, you know, a week where you have five hundred things due, like, no way. So, I guess I'm just still kind of up in the air with that."

and later,

> "I still don't know really if I want to go into academia or go into industry. Um, so I guess it really depends on which of those two but I guess industry I just envision like, 9 to 5, sitting there, solving ... Either doing stats or solving, um, like LP's and then academia I guess kind of similar to what grad school is like, except teaching instead of sitting, being in the class. Teaching, you know, instead and doing research. So, I don't know. I don't really have a very, like, concrete vision."

The absence of clarity in direction is probably related not only to a new realization of the skills required for teaching, but also to the mathematical expectations in the program. Her early professional images of mathematics in the context of her provisional self have a heavy focus on valuing creation of knowledge, communication, problem-solving, breadth, and applications. While her coursework and research feedback both supported valuing multiple approaches, connections in math, and open-ended problems, she addresses her research expectations in the second interview by saying,

> "I haven't really gotten into research for my Master's Project yet. I just found an Advisor like a couple of weeks ago. Um, so I guess really this year has been mainly about course work."

We gain additional insight into her internal conflict from her responses to survey items related to the role of teaching in the professional lives of mathematicians. On the initial survey, her perception was that mathematics professors spent little time thinking about teaching, nor were they expected to spend most of their time on research. By the end of the first semester, those views had changed. She now saw math professors as spending most of their time on research, but also spending time thinking about teaching. She received conflicting messages from faculty and peers about the role of teaching, but overall felt that teaching was valued. During the second interview, she stated,

> "I would say Clemson seems to emphasize the importance of teaching more than I kind of thought grad school would because I thought most schools would ... I don't know it just ... I was under the impression that bigger schools just didn't consider it important. Like I came from a small liberal arts school and it was very important there but they didn't have a graduate program or anything so the main focus of, you know, all the professors was, was teaching ... Um, but we're in, we have this class so that all the first year students have to take that once a week on basically how to teach. So the fact that teaching matters enough for them to run that class kind of surprised me. Um, and I would say that most of the professors that I interact with seem to take teaching pretty seriously. Like, both the ones that I've TA'd for and being in their class, um, and maybe one of two exceptions ... And some of the grad students, I know some grad students who teach their own class and take it very seriously and then I also know some who don't seem to take it seriously at all so I think that's a pretty mixed bag. Um, but I guess overall I would say I was surprised by how important it was considered because I kind of was expecting it to be something that was, was not given high priority at a bigger school."

Drawing only from the second interview, we have the profile in Figure 5.1.4 for who Anna was at the end of the first year in the program. She valued balance and flexibility in her time, enjoyed the skills she had gained to help others with math by explaining clearly and exhibiting didactical skill, and was surprised by the emphasis Clemson placed on good teaching. Although she had received messages from some professors that she should de-emphasize teaching, she felt that the stronger messages were to value teaching, and she now saw teaching as a potential part of her professional identity. She was beginning to explore issues of questioning techniques as a didactical skill to allow deeper knowledge of her students as learners, was questioning the balance of exploration and explanation in a mathematics classroom, and she valued observing other teachers to deepen her own skill.

One year later, at the conclusion of her second year in the graduation program, Anna received an award as an Outstanding Master's student. She is continuing in the program and plans to obtain her Ph.D., although she still has not decided whether she plans to pursue an academic position or an industry position on completion of the degree.

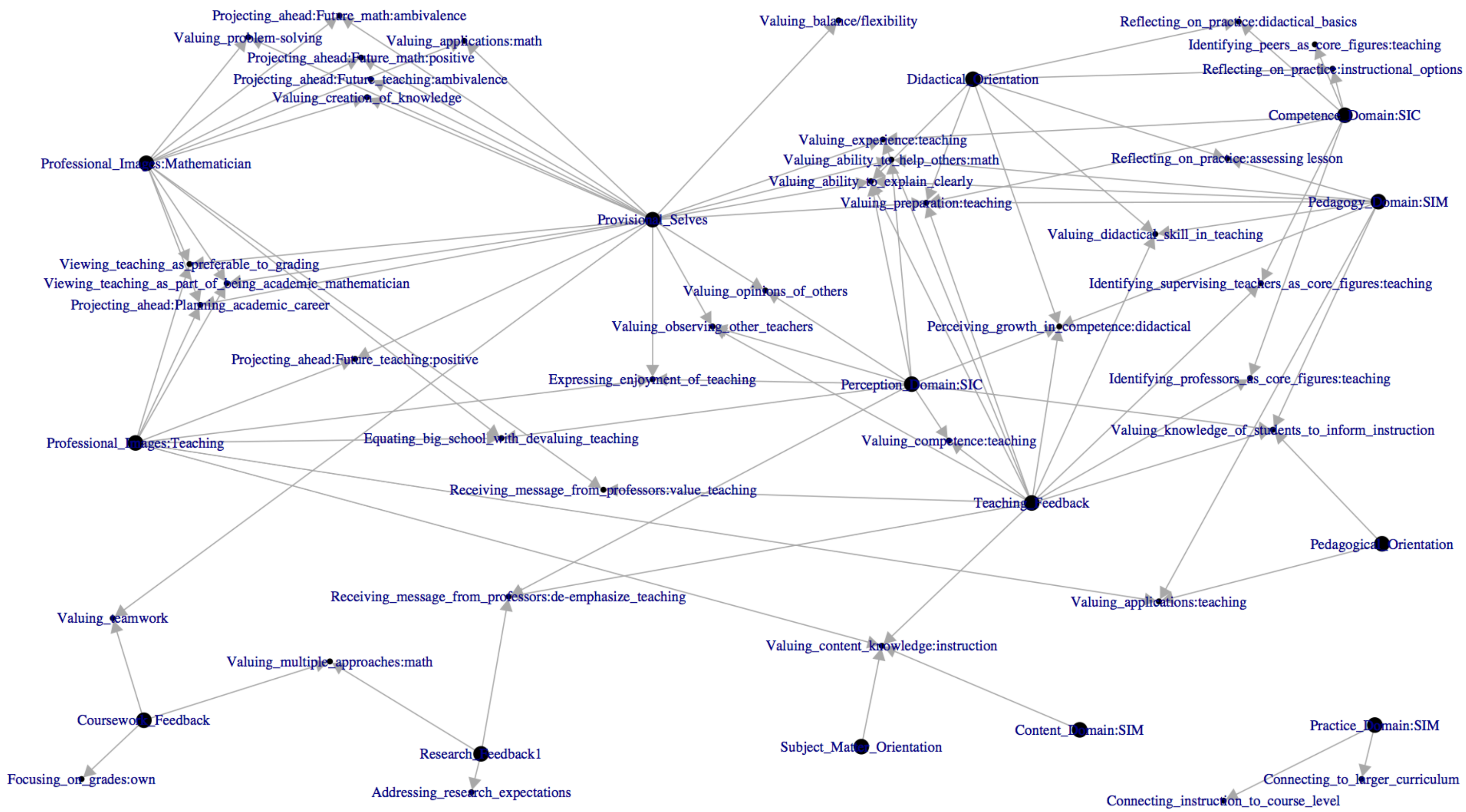
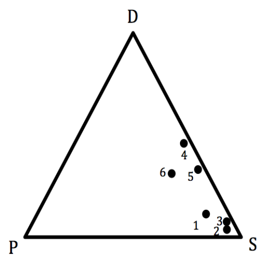


Figure 5.1.4: GTA1's Profile Using All Three Frameworks for Second Interview Data. Anna's profile using all codes applied to Interview 2, showing interrelation between the three theoretical frameworks. Each of the code categories is displayed with a larger vertex size than the codes associated with that category.

5.2 GTA2 (Bill)



GTA 2

Mean Mathematician Identity Change: 0

Mean Teacher Identity Change: -0.151515152

Mean epistemic Change: $+0.28$

	✓	Prompt	Pre-	Post-
Math	✓	I would like to spend less time in school doing mathematics.	Strongly Disagree	Agree
		I will be happy when I am done taking math classes.	Disagree	Agree
Images (Teaching)	✓	The only reason I am teaching is because I have to.	Strongly Disagree	Agree
		Math professors are expected to spend most of their time on research.	Agree	Strongly Disagree
		I would be happy if I never taught math.	Disagree	Agree
		I have a real desire to teach.	Agree	Disagree
		I enjoy talking to other people about teaching.	Agree	Disagree
Beliefs (Teaching)		Time should be spent practicing computational procedures before students are expected to understand the procedures.	Agree	Disagree
		Children should understand the meaning of multiplication and division before they memorize basic math facts.	Disagree	Agree
		Students learn mathematics best by figuring out for themselves the ways to find answers to math problems.	Disagree	Agree
		Children will not understand multiplication and division until they have mastered some basic math facts.	Agree	Disagree

Table 5.2.1: Table of Selected Survey Item Results for GTA2 (Bill). Pre- and post-survey prompts for which Bill's responses changed from Agree to Disagree or vice versa. The survey was forced-choice with four options: Strongly Agree, Agree, Disagree, Strongly Disagree. Items that differed by two steps are indicated by a checkmark (✓) in the '✓' column.

Bill's interest in mathematics began in sixth grade with awareness that he was outperforming his peers and a conclusion that perhaps he had a 'natural gift' for mathematics. He first considered teaching mathematics when he was in high school taking precalculus:

> “I was pretty good at it and there was a couple people that sat right around me and I ... the teacher didn’t really teach. She gave you the homework and let you work on it in class, you could ask questions, and every once in a while she would teach something. So, usually you had to use the book to figure out how to do stuff. And so I was able to help the people who sat around me and I really enjoyed that.”

He actively sought out tutoring opportunities in mathematics throughout high school and college, and derived considerable satisfaction from helping others and from seeing them ‘get it.’ He initially applied to college as an accounting major, but switched to mathematics during the summer before his freshman year, when he sat down to actually choose his first semester courses. His decision to do so was based on a view that he ‘could do better’ than accounting. He felt that since he could succeed at mathematics and many others could not, that was what he should do. Although he had no prior teaching experience or pedagogical coursework, he entered the graduate program with prior tutoring experience and a plan to obtain a Ph.D., work in industry, and then return to academics or pursue teaching at the high school level.

Bill’s previous experiences with tutoring and with mathematics led to a strong didactical and subject-matter orientation. We see this not only in his case identity trajectory, but also in his Beijaard profile using the interview, lesson study, and written reflection data (see Figure 5.1.1). Bill expresses concern over student affect, values that ‘aha moment’ that motivated him from early on, and recognizes a need for meeting individual learner needs.

> [E]veryone is going to have their different styles of learning, and their different speeds, and math doesn’t come as easy to some kids as it does to others, and I just ... I’ve seen teachers get frustrated at students, and then nobody learns anything, and nothing actually gets accomplished. And it’s an easy thing to get frustrated at because it seems so simple to you, but it’s, it’s not when you’re first learning it for the first time, and that’s the reason.

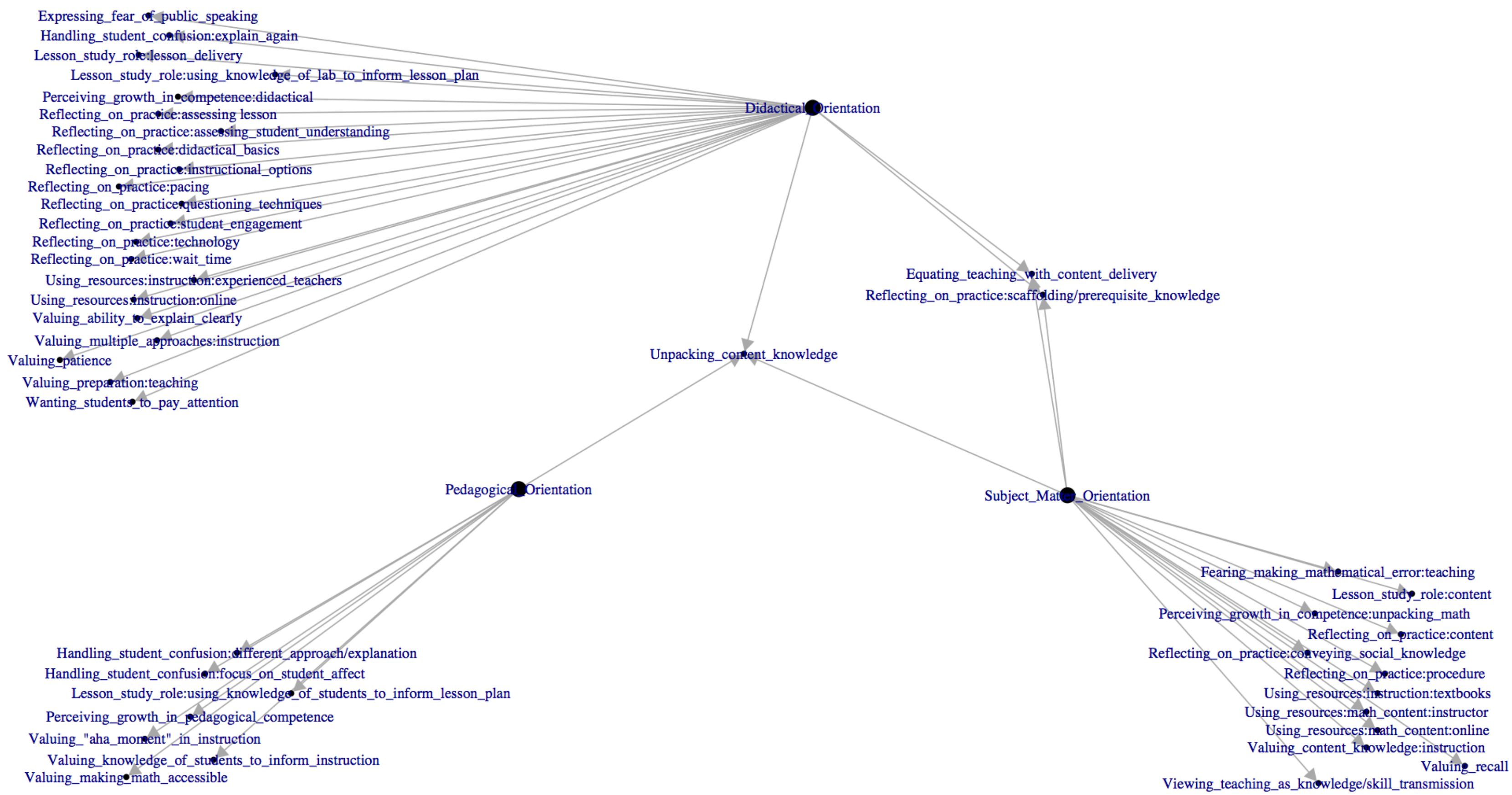


Figure 5.2.1: Bill's profile using codes associated with each of Beijaard et al.'s three orientation categories and incorporating data from both interviews, both lesson studies, and all six written reflections. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the three code categories is displayed with a larger vertex size than the codes associated with that category.

The majority of his reflections and interview comments, however, are directed at didactical and subject-matter issues, often with an underlying level of self-doubt.

> "A girl asked me about the inverse sine of like $\pi/3$ or something, or $-\pi/3$, and I understood how it worked, but I wasn't, I didn't understand that, what the interval was for the inverse sine. That it was from $-\pi/2$ to $\pi/2$ and I was using just the full unit circle at first. And so I had an answer that was correct but they wanted the negative of it, and because of that I like, it just wasn't clicking right away for me, and I ended up, like having to look on my phone to see like what the answer."

> "I have learned to not expect students to understand all material leading up to a certain topic. I used to be nervous and race through examples, but I have slowed down and started asking more questions. In doing so I find that sometimes the new material isn't the problem, but it is one of the foundation pieces giving them trouble ... [T]he semester as a whole was very humbling. There were many times in [precalculus] that a question was asked, and I didn't have the answer. [Participation in the combined course] allowed me to find help and gain a deeper understanding of specific topics. I am much more adept at asking for help or researching solutions now, instead of just trying to plow my way through and maybe not explain something well, or worse, explain it wrong."

The shift towards a balance between didactical and subject-matter expertise is reflected in this writing. Bill's focus is on developing a deeper subject-matter understanding in order to structure lessons and explanations more effectively, and he is doing so within a teaching community of practice. In Figure 5.2.2 we see strong connections to both of the Self-in-Community domains, far outweighing the codings within the Self-in-Mind domains. He is navigating his teaching identity through perceptions of himself and of others within the community, and through negotiated roles and reflection on practice with others, rather than as a primarily internal or individual activity.

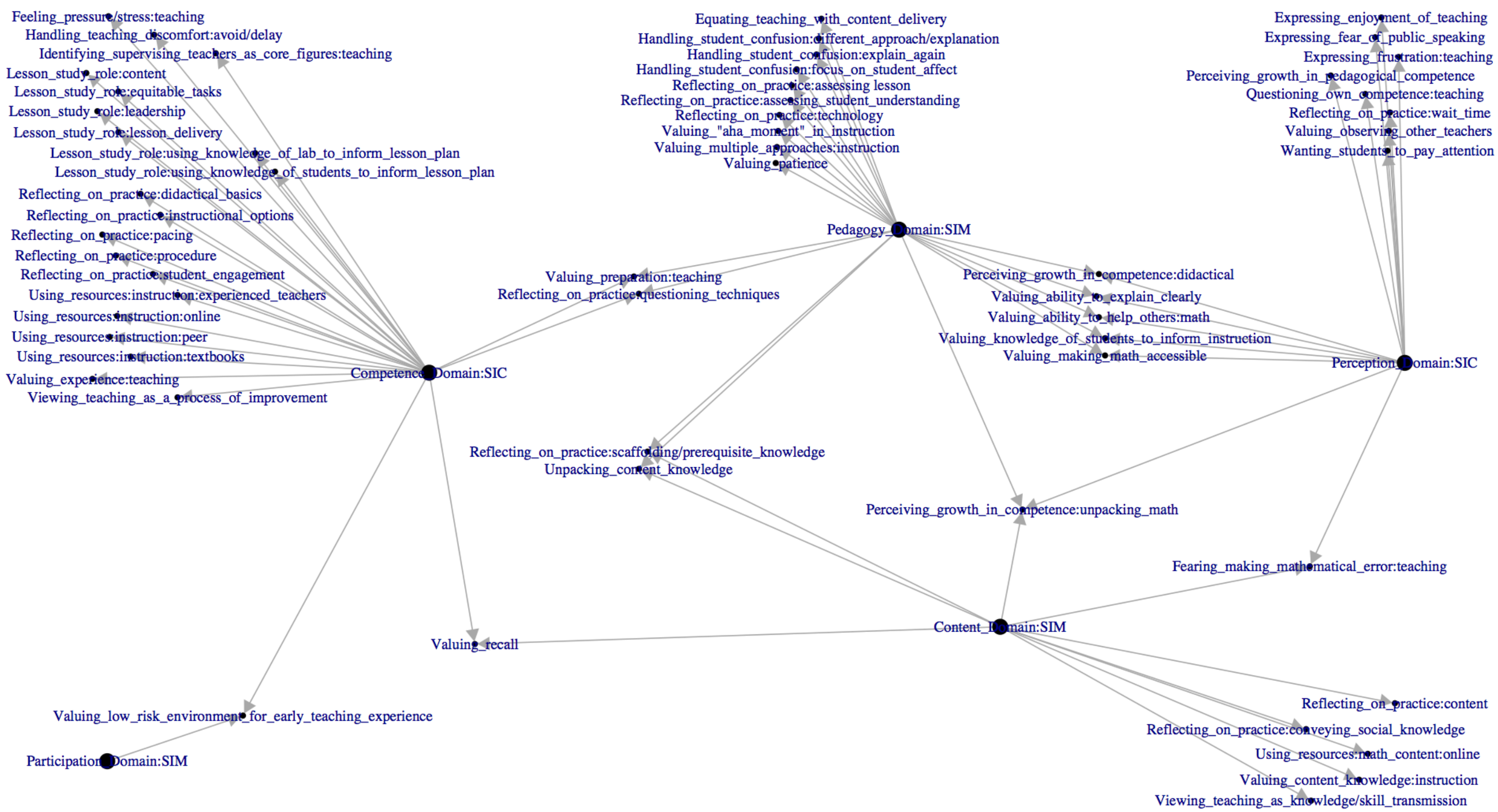


Figure 5.2.2: Bill's profile using codes associated with each of the five categories adapted from Van Zoest & Bohl's framework for teaching identity development, and incorporating data from both interviews, both lesson studies, and all six written reflection prompts. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the five code categories is displayed with a larger vertex size than the codes associated with that category.

While Bill expresses some fear of making mathematical errors in teaching, and some pressure or stress related to teaching, he also strongly values what he views as a 'low-risk' environment in the precalculus course structure.

> "I especially, I have a slight fear of public speaking so I enjoy being able to stand in front of, even just like a group of six or seven kids and teach them a topic just to work on that. And then also, this semester has been fairly low risk, as far as teaching goes ...because in any given point I'm never talking to more than ten students. So the worst case scenario is I might mess up on one little thing for ten kids and then I can come back the next time and explain to them my mistake, or something. So it's, it's a good way to practice this semester because next year I will be teach-, most likely teaching a class of kids. So I'm actually very grateful for the [precalculus] class that I'm in because it gives me the experience at, like a smaller level."

On the other hand, he also perceives his role within a teaching community as temporary, writing in a lesson study reflection,

> "When my group presented I always said I would present, but tried to push the other members to do so instead. The reason for this was because they are all going to be teachers in the near future and this class offers a low risk environment to practice with."

Bill indicated on the post-survey at the end of the first semester that participation in the combined course and teaching precalculus had resulted in no change to views of mathematics or teaching. Indeed, there was very little shift in the subscales. The responses to specific items, however, are quite striking. In particular, by the end of the first semester, he was quite disenchanted with coursework, wishing he spent less time doing math and looking forward to the day he could stop taking math classes. He was equally disenchanted with teaching, viewing it as something he did only because he had to, something he would be happy not doing, and something he no longer enjoyed talking about to others (see Table 5.2.1). These responses are startling, particularly in light of interview comments during the first interview such as,

> "I mean, the whole, this TA I think I lucked out. It's probably one of the best TA's that you could do. Other than maybe if you wanted to teach a class ... because you actually get interaction with the students and although I probably TA just as much as everyone else, it doesn't feel like that, because they're stuck grading, where I get to go to class and actually explain things to kids ..."

The discrepancy between the two data sources demands reconciliation, and we find it in an exchange from the first interview. After Bill indicated that he was really enjoying teaching, the interviewer reminded him that he had previously mentioned a recent change in what he thought he might want to do. Bill replied,

> "Yeah, that's um, that's strictly based on my classes ... Like my math classes, and just how much, how stressful they are at the moment (laughing). Um, yeah, nothing that's, I've ever done teaching wise has ever made me think I wouldn't ever want to do this. It's never been that kind of thought in my head."

His disavowal of teaching, then, was directly driven by his reaction to the coursework and research feedback received within the broader mathematical community of practice. As we see in his profile using Ronfeldt & Grossman's framework (see Figure 5.2.3) he was extremely overwhelmed by the level of difficulty in his courses and by the amount of time required to succeed in those courses.

> "And then this semester the, these past two weeks especially have really scared me a bit because I've, I had a taken home exam that was due today. I had an exam yesterday, homework that was due today and I have two homeworks that were due last week as well ... And so like trying to find the time to do all of that. I mean I finished the take home exam today an hour before it was due."

The feedback he was receiving also chipped away at his self-perception of having a 'natural gift' for mathematics. He compared his own performance and competence to that of his peers at least indirectly:

> "I mean, I just finished ... Umm, in the Math Department we have to take six prep courses, one of them being an Algebra course. And, the amount of, there's homework every week. And the tests are really hard, especially for me. And, so, the amount of time that I had to put into that class compared with my other two classes, it's probably double."

Those experiences led him to make difficult choices to place grade performance over his own mathematical interests.

> "And it was kind of disappointing because, especially the one class, I, I really enjoyed. It was an applied statistics course, time series. And, I mean, the other one was a theory based statistics course, so I, I would have much preferred to focus on those more, but I knew I had to spend as much time as possible on the Algebra, or I wasn't going to pass it, really."

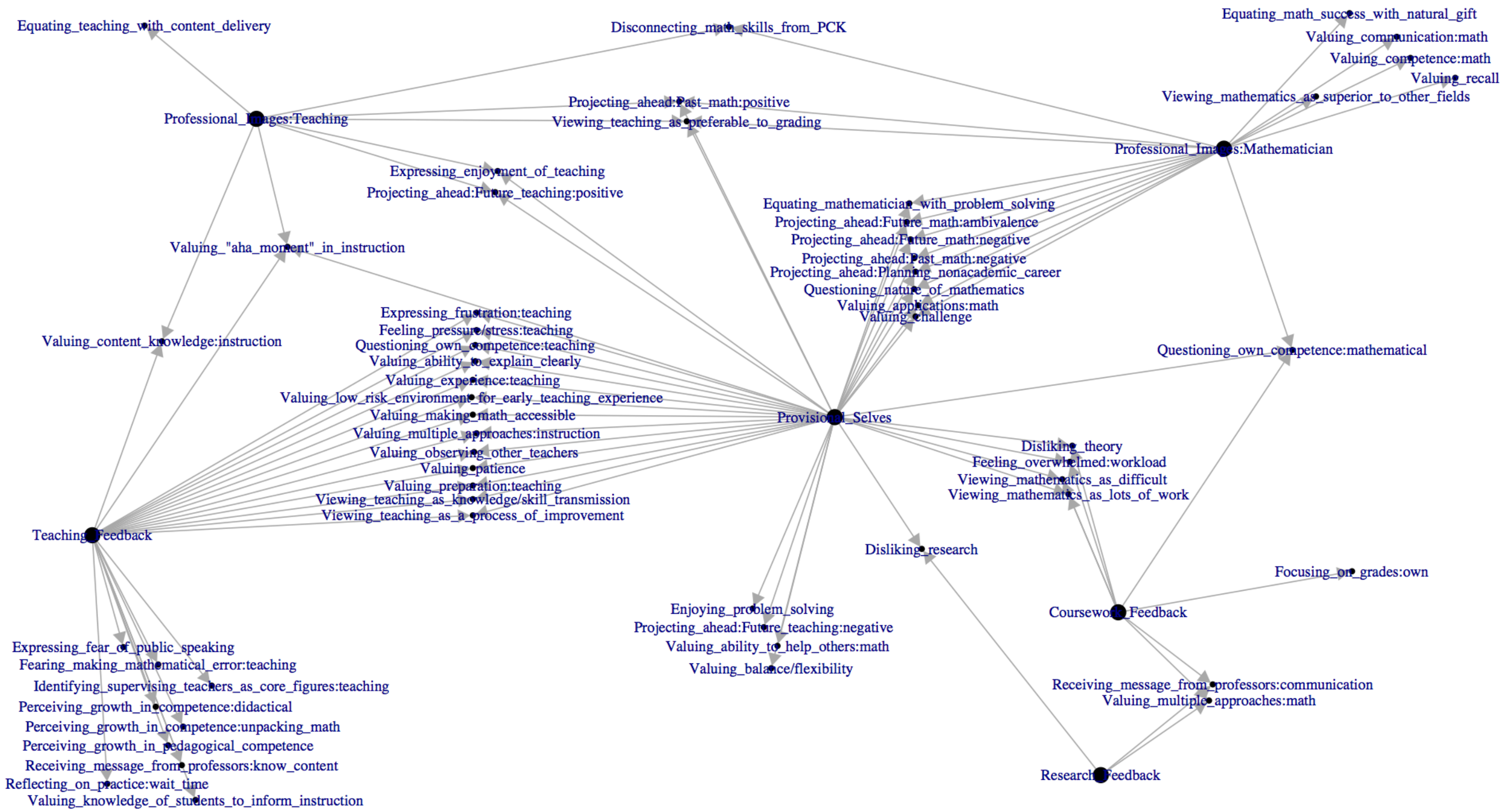


Figure 5.2.3: Bill's profile using codes associated with each of the six categories adapted from Ronfeldt & Grossman's framework for professional identity development, and incorporating data from both interviews, both lesson studies, and all six written reflection prompts. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the six code categories is displayed with a larger vertex size than the codes associated with that category.

By the end of the first year, Bill was seriously questioning his ability to achieve the goals he had brought to graduate school, including his vision of himself teaching. During the second interview, he once again explicitly linked those doubts not to his teaching experiences per se, but rather to the course and research feedback he had received.

> "As of right now though, I don't think I actually want to become an actual teacher ... for awhile I thought I wanted to become a college professor but I don't know if I'm ready to go for a PhD ... So I, it's more of the schooling that's stopping me ... it's not like the lack of enjoyment or something ... It's just, I don't know if I can ..."

By the time of the second interview, Bill had also shifted to an entirely didactical orientation in his teaching. In his full profile using only data from the second interview (see Figure 5.2.4) the Subject-Matter and Pedagogical Orientation categories from Beijaard's framework are entirely absent. He was entirely focused on managing technology effectively in the classroom, adjusting the pacing of a lesson, preparing adequately for delivering instruction, and making use of lesson plans provided by other instructors. He had disavowed both research and theory and saw a future for himself that involved problem-solving in the private sector. He continued to question his own mathematical competence and to feel overwhelmed by the workload and difficulty of the coursework in graduate school.

At the end of the second year in graduate school, Bill completed his Master's degree and accepted a position in the private sector. He currently has no plans to return to teaching at either the secondary or post-secondary level.

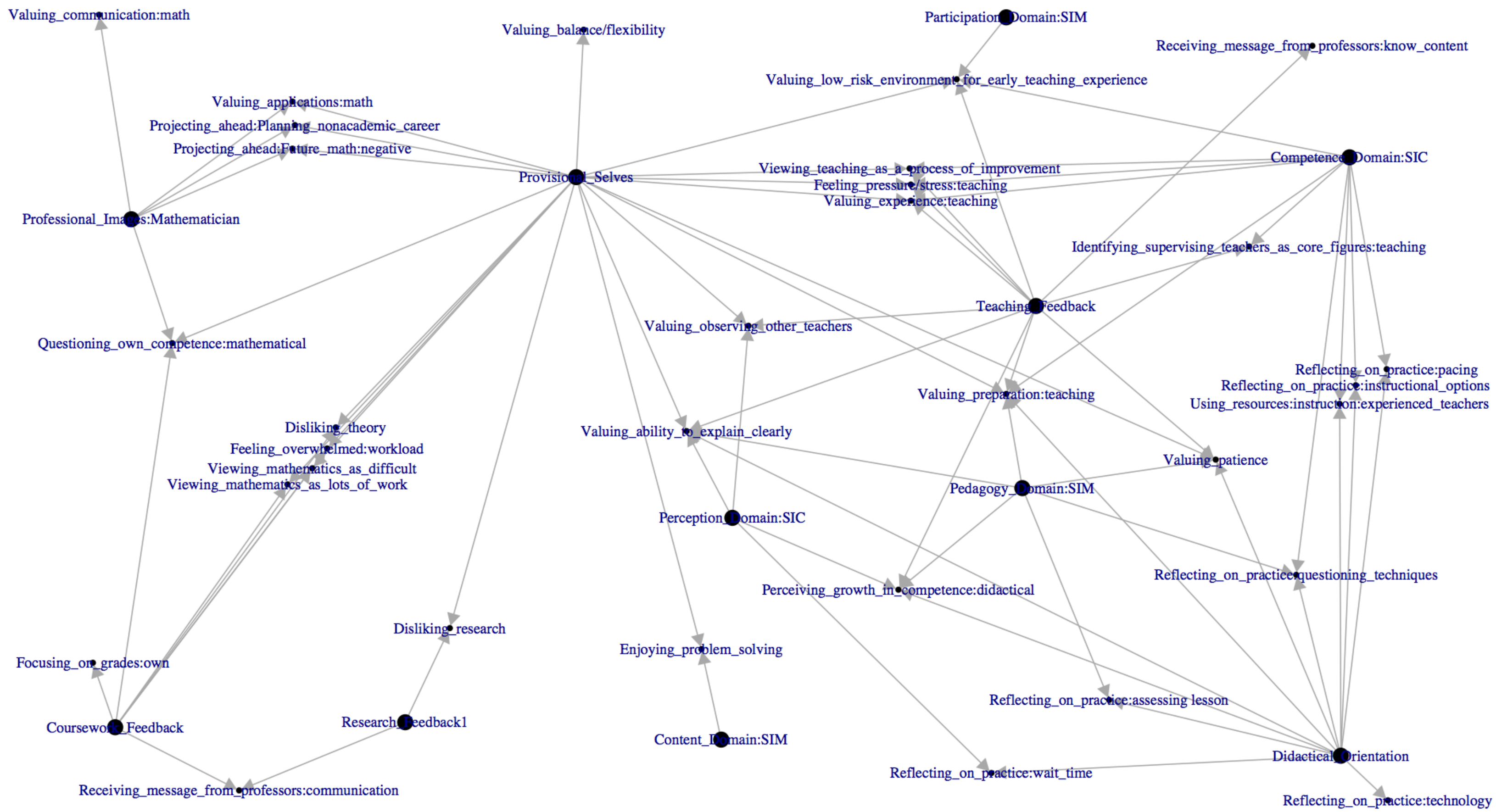
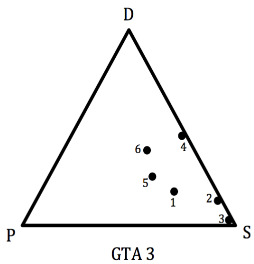


Figure 5.2.4: GTA2's Profile Using All Three Frameworks for Second Interview Data. Bill's profile using all codes applied to Interview 2, showing interrelation between the three theoretical frameworks. Each of the code categories is displayed with a larger vertex size than the code categories associated with that category.

5.3 GTA3 (Cora)



Mean Mathematician Identity Change: -0.139534884

Mean epistemic Change: $+0.2$

Mean Teacher Identity Change: -0.212121212

	✓	**Prompt**	**Pre-**	**Post-**
		I like to do outside reading in mathematics.	Disagree	Agree
		Sometimes I read ahead in my mathematics texts.	Agree	Disagree
Images (Teach)		Math professors are expected to spend most of their time on research.	Disagree	Agree
		Math professors spend very little time thinking about teaching.	Disagree	Agree
		No matter how hard I try, there are some math topics I cannot teach well.	Disagree	Agree
Beliefs (Teaching)	✓	Students should understand computational procedures before they master them.	Strongly Agree	Disagree
		Students should be allowed to invent ways to solve math problems before the teacher demonstrates how to solve the problems.	Disagree	Agree
		Students can figure out ways to solve many math problems without formal instruction.	Disagree	Agree
		Students should solve mathematical problems before they master computational procedures.	Agree	Disagree
		The teacher should demonstrate how to solve math problems before students are allowed to solve problems.	Agree	Disagree
		Children will not understand multiplication and division until they have mastered some basic math facts.	Agree	Disagree

Table 5.3.1: Table of Selected Survey Item Results for GTA3 (Cora). Pre- and post-survey prompts for which Cora's responses changed from Agree to Disagree or vice versa. The survey was forced-choice with four options: Strongly Agree, Agree, Disagree, Strongly Disagree. Items that differed by two steps are indicated by a checkmark (✓) in the '✓' column.

Cora knew exactly why she chose to pursue mathematics: she had a high school geometry teacher who made math "stupid fun" and she wanted to be like that teacher. That particular class had a strong impact not only on Cora's views of teaching and of mathematics, but also on her mathematical self-efficacy.

> "Oh, she just made it stupid fun. Uh, I've always liked math, and I al-, was always good at math. S-. But, um, she really stretched me in, uh, it was geometry. Real-, She really stretched me in geometry past, some of my classmates, because she was like, 'Oh, you're-. You could go farther,' and she made me go farther. [laughs] ... And also just her-. Um, the way she like interacted with students. Yes, they respected her. Yes, they wanted to obey her, but she also cared about them, and they knew that. And so, she had students always come to her, um, wanting, you know, her advice ... Oh, and another thing she did: We actually-. Um, so it was geometry, you know, you do a lot of geometric proofs, and she had us, for one of the projects, we had to come up with our own thing to prove. Um, so, like, the classic SAS properties or ASA congruence stuff like that. And then we had to come up with our own figure, and come up with some weird thing in it that, given such and such we can get to that. And then that became part of our next test. So, since we create it we could get the answer right, and then everyone else in the class had to figure it out."

Although Cora completed an undergraduate degree in mathematics education, she did not complete her teaching certification but chose instead to apply to graduate school. Her older brother had just received his Ph.D. in mathematics, and her older sister had just passed her qualifying examinations for a Ph.D. in mathematics. At the time of the first interview Cora acknowledged their influence to attend graduate school. Their experiences may also account in part for her views of what skills are necessary to succeed in mathematics:

> "They really have to be able to view a subject and really recall, like, everything that pertains, so that you can, hopefully, pull everything together. And just the big picture is huge, which a lot of times, when we're studying little theorems and stuff, we get bogged down on the little minute details because we're trying to understand that and we lose the huge picture. Which my sister's always, like, um, admonishing me when I'm studying or something and I'm like, 'I don't get this!' She's like, 'Well, get the big picture at least.' And, so, the big picture's huge, it's just that you can actually tie everything together. But you can't get the huge picture of mathematics because of all the different fields, but maybe you could get, like, the big picture of algebra, maybe. "

The influence and example of older siblings having successfully navigated graduate school in mathematics appears to have been a double-edged sword for Cora. While she had family support and often turned to her sister for assistance and mentoring, her focus on teaching mathematics did not fit their path. She could not reconcile the role she wanted to play as a teacher in the mathematical world with her view of what it meant to be a mathematician.

> "To me a mathematician is one who focuses on math, as in they're trying to go further

> and make math expand, add their own discoveries to it so that, further down the road, someone can look up their paper and be like, ‘Oh, look at this. This has been done, now we can go even farther,’ and create new math or discover new math, whatever word you want to use. I don’t know if I really wanna be that person. But, if you define mathematician as the one who, performs the math (laughs), then, I don’t know. I’ve always loved math, and I’ve always wanted-, thought it was just fun to do math. So-. And I guess maybe at the time I wanted to go to grad school, it could possibly go in there. Maybe. Not necessarily.”

Although Cora indicated at the start of the first semester that she planned to pursue a Ph.D., by the time of the first interview she was questioning that goal openly. By the time of the second interview, she made no mention of her siblings’ influence, nor did she reference her previous objective of getting a Ph.D., saying instead:

> “When I was looking last fall- or last spring, as I was looking towards graduation, and like, Masters or teach right away, um, God opened the door for me to get my Masters and I wasn’t- I didn’t feel like I was prepared to teach at that point. I mean, I had all the classes and courses and supposedly, you know, my degree stated, and I had the certification to say, ‘Oh, I can teach,’ but personally, I wasn’t prepared.”

Despite the shift in her degree intentions, Cora’s enjoyment of mathematics, and her confidence with the subject matter for teaching, were clear throughout the entire year. Many of her interview excerpts revolved around specific, detailed recollections of interactions with students, told in first person as a recounted dialogue. She openly acknowledged both her enthusiasm for mathematics and teaching, along with her perceptions of how students perceived that enthusiasm.

> “When I feel confident about something, I get really excited, and the students think I’m really weird, because I’m trying to get it across to them, and I’m really excited.”

> “Or I’ll be like, ‘Guys, this topic is super fun, why aren’t you doing anything?’ And they’re like, ‘It’s not fun.’ I was like, ‘Yes, it is. Look at this. Look how this ties in with this.’ And they’re like, ‘Okay, that is kind of cool.’ I’m like, ‘Exactly!’ (laughs)”

Unlike Anna and Bill, Cora had extensive coursework in pedagogy and instructional theory during her undergraduate studies, and she indicated ‘Strongly Agree’ in response to the initial survey prompts ‘I think of myself as a teacher’ and ‘I have a real desire to teach.’ Using only data from the first interview, her Beijaard identity is heavily weighted toward the subject-matter/pedagogical edge of the triangle (see Figure 5.3.1).

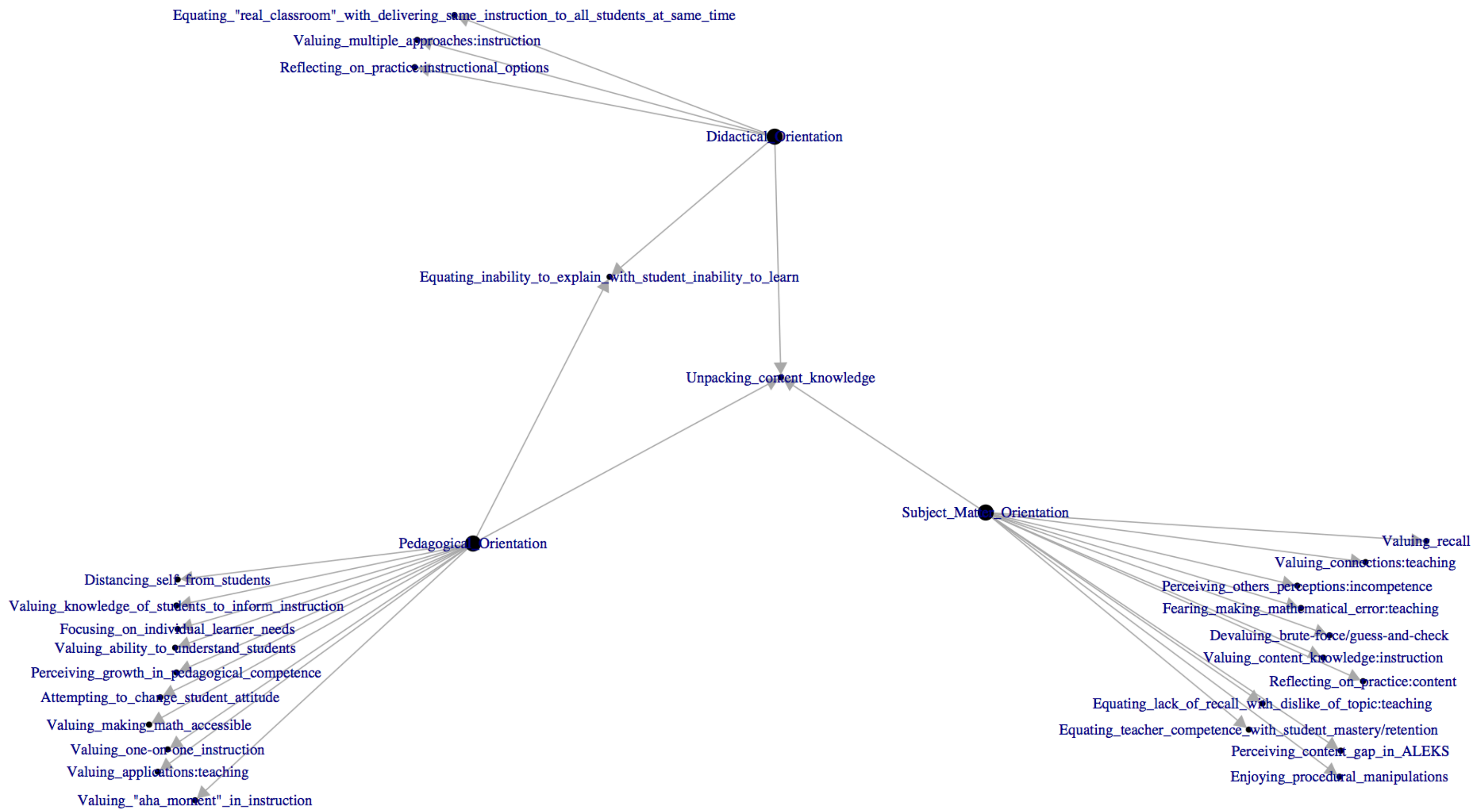


Figure 5.3.1: Cora's profile using codes associated with each of Beijaard et al.'s three orientation categories and incorporating data from the **first interview only**. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the three code categories is displayed with a larger vertex size than the codes associated with that category.

Like Anna and Bill, Cora started out contributing to case discussions in each area of expertise, but then retreated to the subject-matter expertise corner. The retreat was temporary in her case, though, and she slowly resumed making contributions from multiple vantages, with an increasingly central location in the identity triangle. Using only data from the second interview, her Beijaard identity is weighted toward the subject-matter pole, but balanced between didactical and pedagogical (see Figure 5.3.2). Taking all of the data sources as a whole, hers is the most balanced Beijaard identity of the four participants (see Figure 5.3.3).

Despite a slight shift away from a teaching identity, away from a mathematician identity, and towards a more constructivist view of knowledge and mathematics as measured by the survey, Cora indicated no self-perceived changes in any of those aspects. However, her views of what was expected from math professors with respect to teaching did change, as did her own teaching self-efficacy. By the end of the first semester, she saw math professors as focused primarily on research and spending little time thinking about teaching, and she felt that perhaps there were some topics in mathematics that she would not be able to teach well despite effort (see Table 5.3.1).

Another characteristic of Cora that was very evident from the data was her concern with how she was perceived by others. She expressed considerable concern with being liked and respected by her students, perhaps reflecting her desire to emulate the teacher she liked and respected in high school.

> "Well, if I don't feel confident then it's really bad, because you know, you're second guessing yourself. You're like, 'Uh, wait. Isn't this-? No.' And then you're quickly trying to do it in your head to double-check your answer. (laughs) And you're like, 'Oh, stink. Wait. Hold on just a second while I think about this question. Ummm ... I think it's this. I'm pretty sure.' And then the students get the idea of, 'She does not know what she's talking about.' And it's like, 'I'm sorry. I really do, but I don't at this moment.' Like, 'I've done this, I promise. Maybe it was eight years ago, but that's okay.' "

That desire to be liked and respected threaded through her desire to have strong content mastery and to develop her pedagogical skills.

> "Yeah, so just knowing your students is really important, and knowing your material, obviously, is super important. Because no one likes a teacher who doesn't know what they're talking about. (laughs) And no one respects the teacher who does not know what they're talking about."

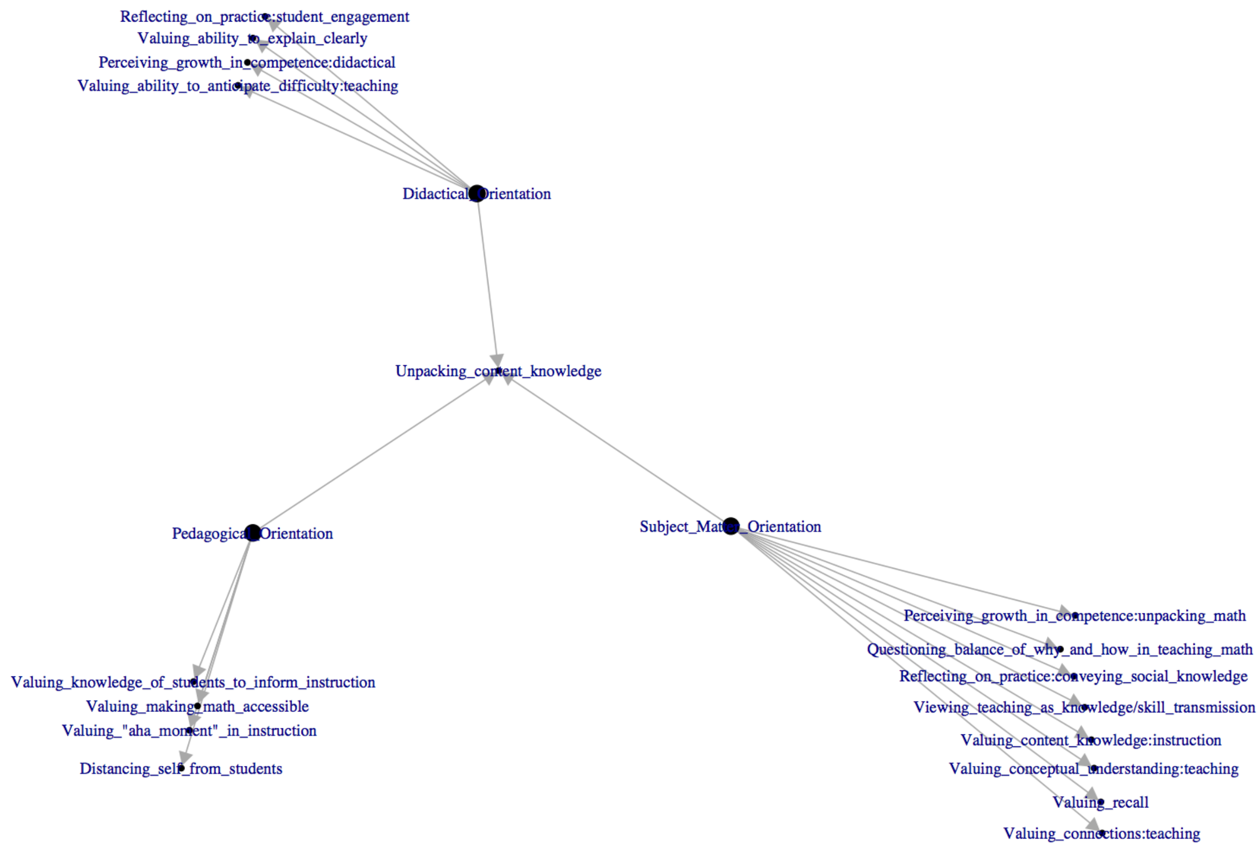


Figure 5.3.2: Cora's profile using codes associated with each of Beijaard et al.'s three orientation categories and incorporating data from the **second interview only**. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the three code categories is displayed with a larger vertex size than the codes associated with that category.

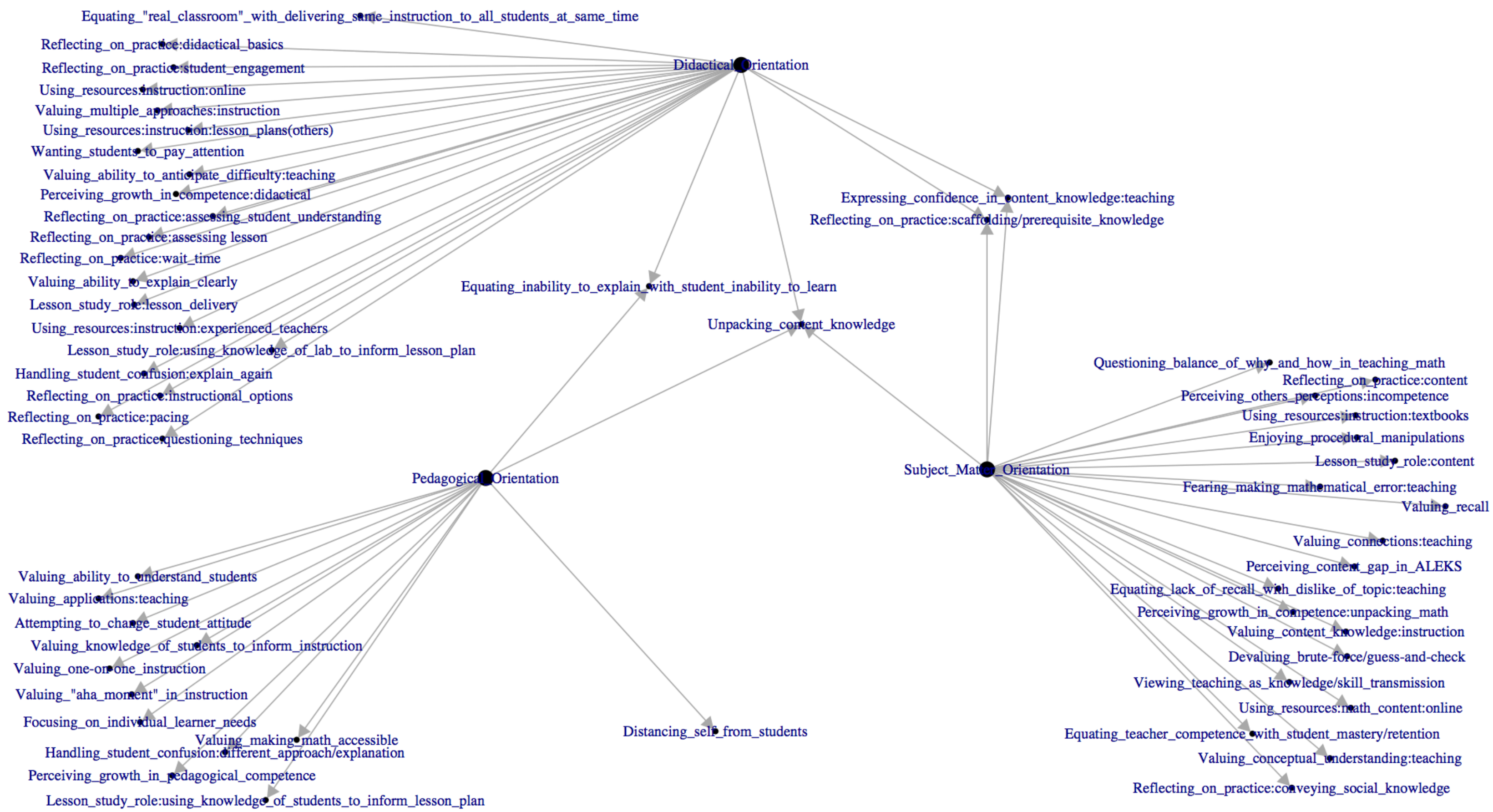


Figure 5.3.3: Cora's profile using codes associated with each of Beijaard et al.'s three orientation categories and incorporating data from both interviews, both lesson studies, and all six written reflections. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the three code categories is displayed with a larger vertex size than the codes associated with that category.

Like Bill, Cora placed strong value on the precalculus teaching experience, although for her it was not the low-risk aspect but rather the ability to work one-on-one with students that she valued. She specifically mentioned both the precalculus teaching experience and the case studies as valuable components for developing pedagogical content knowledge.

> "I'm always trying to understand how students think and the misconceptions and common mistakes they have. By reading the case studies and watching/teaching/tutoring/grading for [precalculus], I have learned more misconceptions and mistakes. I can teach kids the theory and the correct way to do stuff, but if they are consistently making more than simple errors, I need to know how to help them. Hopefully, the more experience I get the easier it is."

Cora's profile using Van Zoest & Bohl's framework (see Figure 5.3.4) helps convey just how much and in how many ways she thought about mathematics and the teaching of mathematics. There are extensive codes associated with each category; many of those codes contain multiple codings. Some of the more interesting codes from her profile include "Equating 'real' classroom with delivering same instruction to all students at same time' and 'Equating inability to explain with student inability to learn.'

Her survey responses help reconcile the juxtaposition of those codes with 'Focusing on individual learner needs' and 'Valuing ability to understand students'. Cora responded 'Strongly Agree' to items such as "Students should be told to solve problems the way the teacher has taught them" and "To be successful in mathematics, a student must be a good listener." Similar responses across that subscale of the survey support the conclusion that Cora saw her role as a teacher as adjusting mathematical representations to present mathematical content in multiple ways. In the first interview, when asked what she viewed as the most important skill to develop as a math teacher, she responded:

> "Just trying to understand how your students think through something, um, because everyone learns differently, everyone sees things differently when they read something for the first time. Everyone will kind of interpret it, uh, slightly differently. Yes, they'll get the-, they might get the meaning but they'll come about that meaning in their own way. Um, some people will have to draw out a picture in order to get there or if it's really abstract, they might have to make a really concrete example to get there. And so, as a math teacher you have to know, what are-, the tendencies are of your students, and there isn't a one thing that math teachers have to do in order to get there. They have to actually pay attention to what their students do throughout."

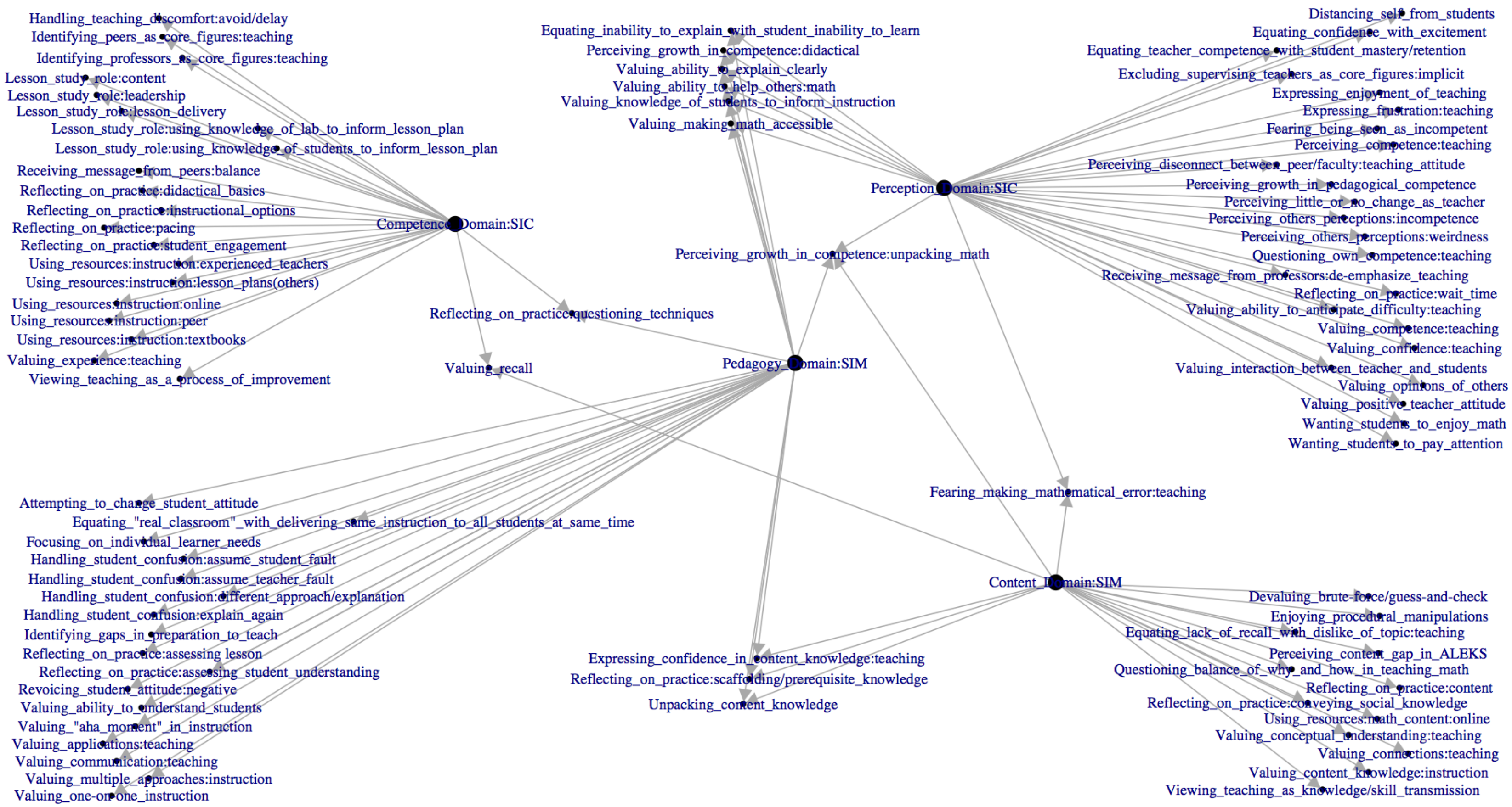


Figure 5.3.4: Cora's profile using codes associated with each of the five categories adapted from Van Zoest & Bohl's framework for teaching identity development, and incorporating data from both interviews, both lesson studies, and all six written reflection prompts. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the five code categories is displayed with a larger vertex size than the codes associated with that category.

Although Cora indicated on her post-survey that she didn't perceive much change in how she viewed teaching or mathematics, she also acknowledged in the second interview:

> "Yeah. I don't know for sure if [my view of teaching] changed or evolved. It probably has, but, you know, when you're in on top of something, you know, if you're with someone every single day, you're not seeing the changes necessarily. Where, so like with myself, I don't see the changes, but if someone else maybe saw something and then they don't see me again, they'll see like, 'Whoa, did you know like this whole thing has changed your life?' "

The data did indeed reveal many changes in her views of teaching, and not just in the identity trajectory from the case arcs, the changes in her Beijaard identity between the first and second interviews, or her views of how teaching fits into the identity of an academic mathematician. She was also reconsidering the role and nature of instruction, as evidenced by shifts in her responses to prompts such as 'Students can figure out ways to solve many math problems before the teacher demonstrates how to solve the problems.' (See Table 5.3.1.) Most of the case arcs discussed in the seminar were presented from a constructivist perspective. We speculate that seminar conversations around those cases, coupled with one-on-one interactions in the precalculus classroom, brought those issues into question for Cora. Her second interview provided evidence of shifts in attitude, although no direct evidence to support attribution of those shifts specifically to case arcs or precalculus interactions. Our speculation as to cause comes in part from the absence of other explanatory factors in the second semester teaching experience.

Cora was the only participant who continued with a teaching assignment to Precalculus in the second semester. During the first semester, she placed value on the one-on-one instructional experience, despite dismissing the course as not a 'real classroom' since it didn't involve delivering the same content to all students at the same time. The value she perceived in the first semester was in refreshing her content knowledge, practicing didactical skills, developing additional methods for presenting content, and working with peers in the seminar setting. In the second semester, she began to reflect instead on the nature of the mathematics she was teaching, and to derive a different type of satisfaction from her interactions with students.

> "I think in the fall, uh, I, as I was working with students, I would try and do, er- sometimes I would do a lot less of the theory, or the background behind it, and just do the problem for them. And this semester I've tried to do a little more of the background of the problem, and an explanation of why this works. Um, maybe not from like the

> ground up, like I did with one girl where I basically proved it to her, but I have done a lot more of the expla- like a deeper understanding, or a- aimed at deeper understanding this semester. ... I think it's more fun to do that, (laughter) then just to give them a how-to. Because they can get a how-to from anybody, and a lot of people give that to them. Or online, they can find the how-to."

Interestingly, when asked in the second interview who had influenced her views of teaching, Cora acknowledged only full professors and her graduate student peers. On probing, she confirmed that she saw no other influences. While we find that a bit perplexing given her regular interactions with the researcher in a teaching setting over the course of two full semesters, it simultaneously relieves some concern about researcher bias and also gives us insight into how she perceives her community of practice (see Figure 5.3.5).

Not surprisingly, Cora came in with strong professional images both of teachers and of mathematicians, and considerable overlap between those images and her provisional selves. Her experiences in graduate school supported several of those early teaching images and the feedback she got from teaching settings had extensive impact on her provisional selves. Some of that feedback included aspects of stress, such as fearing making errors or being seen as incompetent. Much more of it, though, was focused on how she saw herself as a teacher, including valuing connections and clear explanations, knowing her students, and being able to anticipate difficulty. The feedback she received from her coursework and research settings, on the other hand, centered around frustration and difficulty. Like Bill, she began questioning her own mathematical competence.

Although her peers were encouraging her to find balance between research, coursework, and teaching expectations, her professors were encouraging her to de-emphasize teaching. Her full profile at the time of the second interview (see Figure 5.3.6) shows the complexity of the conflicting messages she is receiving, but we note that the majority of her Provisional Selves connections to other categories involve codes associated with teaching.

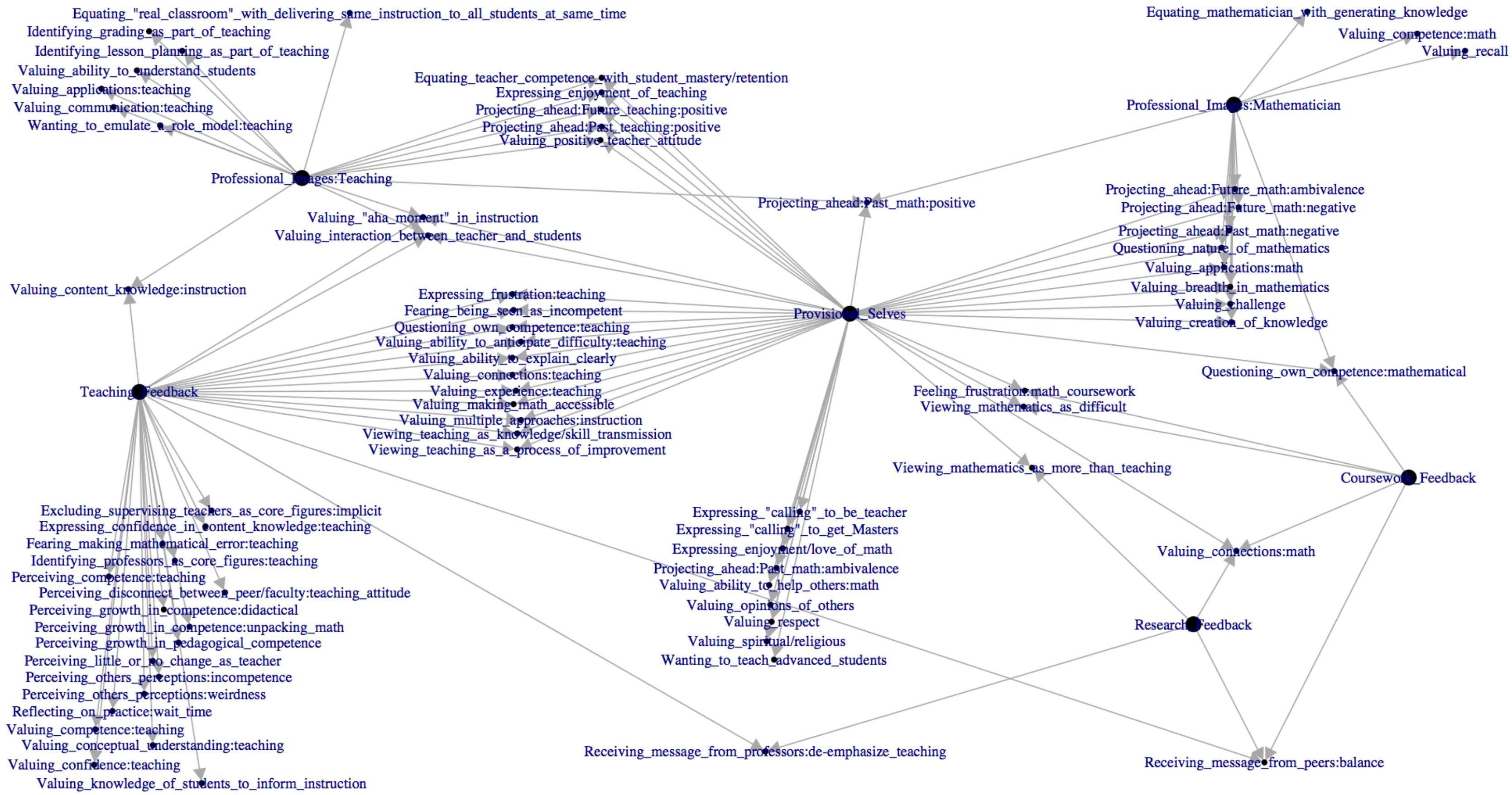


Figure 5.3.5: Cora's profile using codes associated with each of the six categories adapted from Ronfeldt & Grossman's framework for professional identity development, and incorporating data from both interviews, both lesson studies, and all six written reflection prompts. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the six code categories is displayed with a larger vertex size than the codes associated with that category.

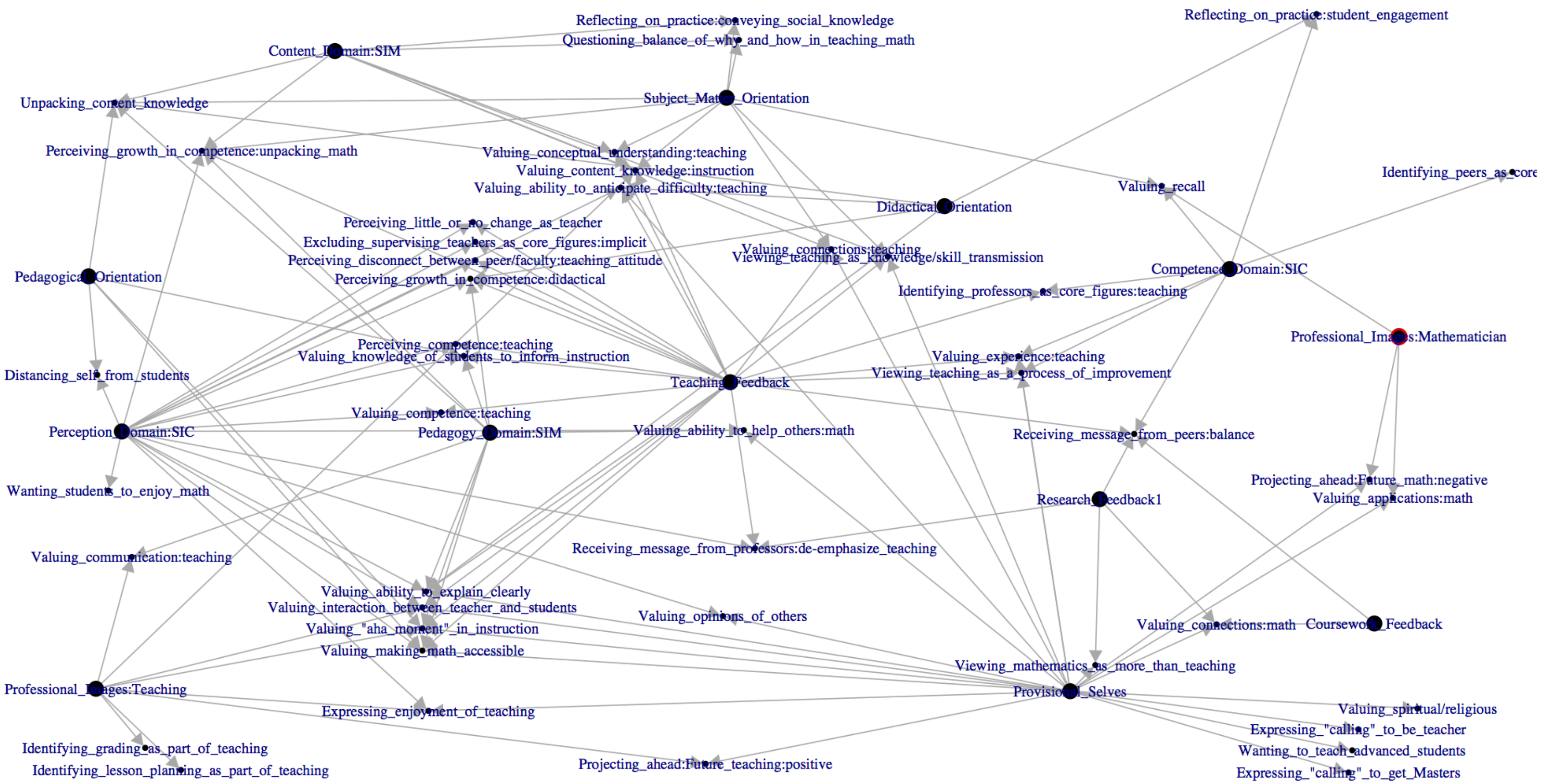
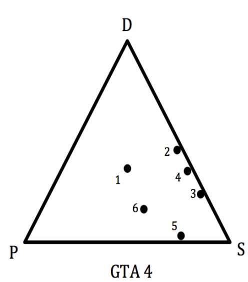


Figure 5.3.6: GTA3's Profile Using All Three Frameworks for Second Interview Data. Cora's profile using all codes applied to Interview 2, showing interrelation between the three theoretical frameworks. Each of the code categories is displayed with a larger vertex size than the codes associated with that category.

As a first solo teaching experience, Cora was assigned as instructor of record for a 45-student section of Long Calculus I. Student performance in her section exceeded that of first-time GTAs without the combined course and seminar experience, but fell below that of Anna, and of experienced GTAs and lecturers. A brief hallway interaction between the researcher and Cora at the end of her first semester solo teaching brought out that she was very dissatisfied with how the semester had gone and with how her students had done. It is worth noting that instructors of coordinated courses are provided with statistical summaries of their students' performance relative to the overall mean. Although Cora's students outperformed several sections, she did not have access to that information and knew only that they had underperformed the mean.

In light of the importance she placed on others' opinions of her competence, and of the fact that she equated instructor competence with student performance, we speculate that this was a significant blow to her sense of self within the community of practice. At the end of the second year in the program, Cora completed her Masters degree in mathematics and accepted a teaching position at a small, private (religion-centered) high school.

5.4 GTA4 (Dave)



Mean Mathematician Identity Change: -0.046511628

Mean epistemic Change: $+0.4$

Mean Teacher Identity Change: -0.424242424

	✓	Prompt	Pre-	Post-
Images (Math)	✓	Most of the ideas in mathematics aren't very useful.	Agree	Strongly Disagree
		My mathematics teachers present material in a clear way.	Agree	Disagree
		I often think, "I can't do it," when a mathematics problem seems hard.	Disagree	Agree

Continued on next page

Table 5.4.1 – continued from previous page				
	✓	**Prompt**	**Pre-**	**Post-**
Images (Teaching)	✓	No matter how hard I try, there are some math topics I cannot teach well.	Strongly Agree	Disagree
	✓	Math professors are expected to spend most of their time on research.	Strongly Disagree	Agree
		Math professors spend very little time thinking about teaching.	Disagree	Agree
		My mathematics professors don't seem to enjoy teaching.	Disagree	Agree
Beliefs (Teaching)		Students should solve mathematical problems before they master computational procedures.	Disagree	Agree
		Children should understand the meaning of multiplication and division before they memorize basic math facts.	Disagree	Agree
		Students should understand computational procedures before they master them.	Disagree	Agree
		Children will not understand multiplication and division until they have mastered some basic math facts.	Agree	Disagree

Table 5.4.1: Table of Selected Survey Item Results for GTA4 (Dave). Pre- and post-survey prompts for which Dave's responses changed from Agree to Disagree or vice versa. The survey was forced-choice with four options: Strongly Agree, Agree, Disagree, Strongly Disagree. Items that differed by two steps are indicated by a checkmark (✓) in the '✓' column.

Like Bill, Dave's first sense of wanting to teach mathematics came in elementary school, when he discovered he could help his classmates, in particular by putting mathematical concepts in more accessible terms.

> "So, um, I was in like somewhere in middle school- I think somewhere in fifth grade, but um ... The girl next to me she didn't like math at all and she kind of was always like, lean over to me and be like, "I have no idea what she's saying." And I'll be like, "Oh, all she's saying is this." And kind of saying it in more general terms without like the uh, the math vocabulary I guess and she'd be like, "Oh I get it now." That's when I was like kind of like, "Oh, this is kind of fun."

Like Cora, Dave completed his undergraduate degree in secondary mathematics education, but unlike her, he completed his student teaching and obtained his teaching credential. It was during the student teaching that he decided he wanted more of an age gap between him and his students, so he decided to go to graduate school and return to teaching later. He enrolled at Clemson mid-year and took undergraduate mathematics coursework to remedy deficiencies in his mathematical preparation prior to starting graduate coursework. His was a provisional admission, and he had to re-apply for full admission after the spring semester. Even then, he did not identify as being or becoming a mathematician, but rather saw obtaining a Master's as a filler until he returned to teaching. Dave is the only one of the four participants who began the graduate program anticipating a terminal Master's rather than a Ph.D. He is also the only one who had taken mathematics courses at Clemson during a prior regular semester, but they were at the undergraduate level rather than the graduate level. He did not participate in the summer bridge program with Anna and Bill.

During his spring semester as a provisional student, Dave tutored 25 hours per week for the athletic program. He worked one-on-one with athletes, and took considerable pride in playing the same role he remembers from fifth grade: making math accessible by putting it in more comfortable terms. During the first interview, he identified that ability as the most important one for a teacher to develop, connecting it both to his tutoring experience and to his student teaching.

> "And so, I thought of different ways to explain it like if kids had trouble with negatives, like negative numbers or subtracting two negative numbers um ... really into like money, because like you said, everyone knows money so you could say like, "You have four dollars and they give you four more. Four plus four is eight." They know that. Or like if you owe me four dollars and now you owe me another ten, now you owe me fourteen, so you're fourteen in the hole, and they could understand that a lot better than just looking at a paper with just negative four plus negative ten or any combination of negatives and positives, so just trying to relate it to them is a big thing too."

Not surprisingly given their similar backgrounds, Dave's identity trajectory from the case discussions is similar to Cora's. An initial willingness to contribute to discussions in all three arenas of expertise quickly changed to a retreat to the subject-matter/didactical axis. In Dave's case, though, the identity location at the end of the semester was less central than at the beginning of the semester, and more closely anchored to the subject-matter corner of the triangle. In his Beijaard identity drawn from reflective writings and interviews (see Figure 5.4.1), on the other hand, we see a heavy weight towards the didactical pole. He valued multiple instructional approaches and making math accessible, but most of the instructional adaptations he discussed were situated in tutoring experiences, rather than in classroom settings.

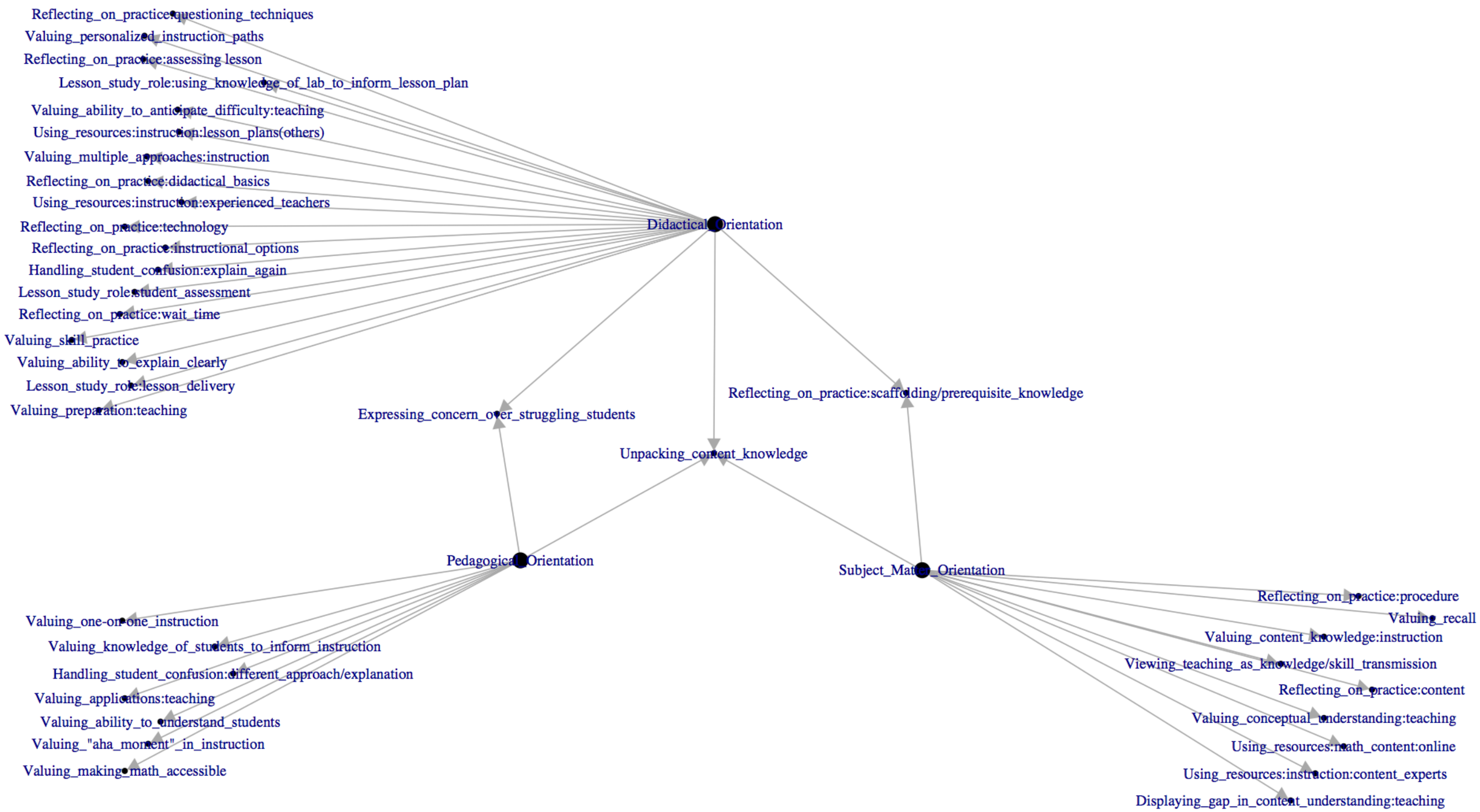


Figure 5.4.1: Dave's profile using codes associated with each of Beijaard et al.'s three orientation categories and incorporating data from both interviews, both lesson studies, and all six written reflections. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the three code categories is displayed with a larger vertex size than the codes associated with that category.

Within the lesson study teams, Dave viewed himself as a leader and expert voice. He contributed his knowledge of the lab setting and students to help structure a suitable lesson plan and was always present either to deliver the lesson or observe a teammate delivering the lesson. in reflecting on group discussions, he wrote,

> "I tried to allow other group members to comment first on the positive/negatives of the lesson and chimed in when I agreed or thought something needed to be added to the discussion."

Dave's self-perception as a leader and as an expert in the seminar setting are reflected in his profile using Van Zoest & Bohl's framework (see Figure 5.4.2). His Self-in-Community domains, particularly the Competence Domain, reflect the fact that he was actively engaged in reflecting on practice, negotiating roles in instruction, in assessing lesson outcomes within the teaching community of practice established in the seminar course.

Dave echoed Bill and Cora's perceptions of the value of the Precalculus teaching experience. In his case, however, he placed the primary value on the opportunity to practice giving the same small lesson multiple times to refine the delivery. He was particularly excited about the ALEKS® instructional materials and the potential for implementing those in a high school setting to allow for more individualized instruction within a larger class setting. It is interesting to note, though, that even by the time of the first interview he appeared to be questioning his career plans; he was no longer necessarily planning to return to high school teaching.

> "[T]hey could do whatever they could on their own. Press the explain button. Maybe they could get it from a program explaining it, but then in the labs when they came in, if there's something they were really, really stuck on, they could get personalized help from us. Whether it's working another problem or building their own skills that they needed that they didn't have in a problem. So, when I first saw it, that's like the first think I thought of and I didn't- I asked her, I was like, "Do they have this at the high school level?" And she said, "Yeah." So if I ever did go back and teach at a high school level um, I would see if they knew anything about it, or if it was something that might interest them."

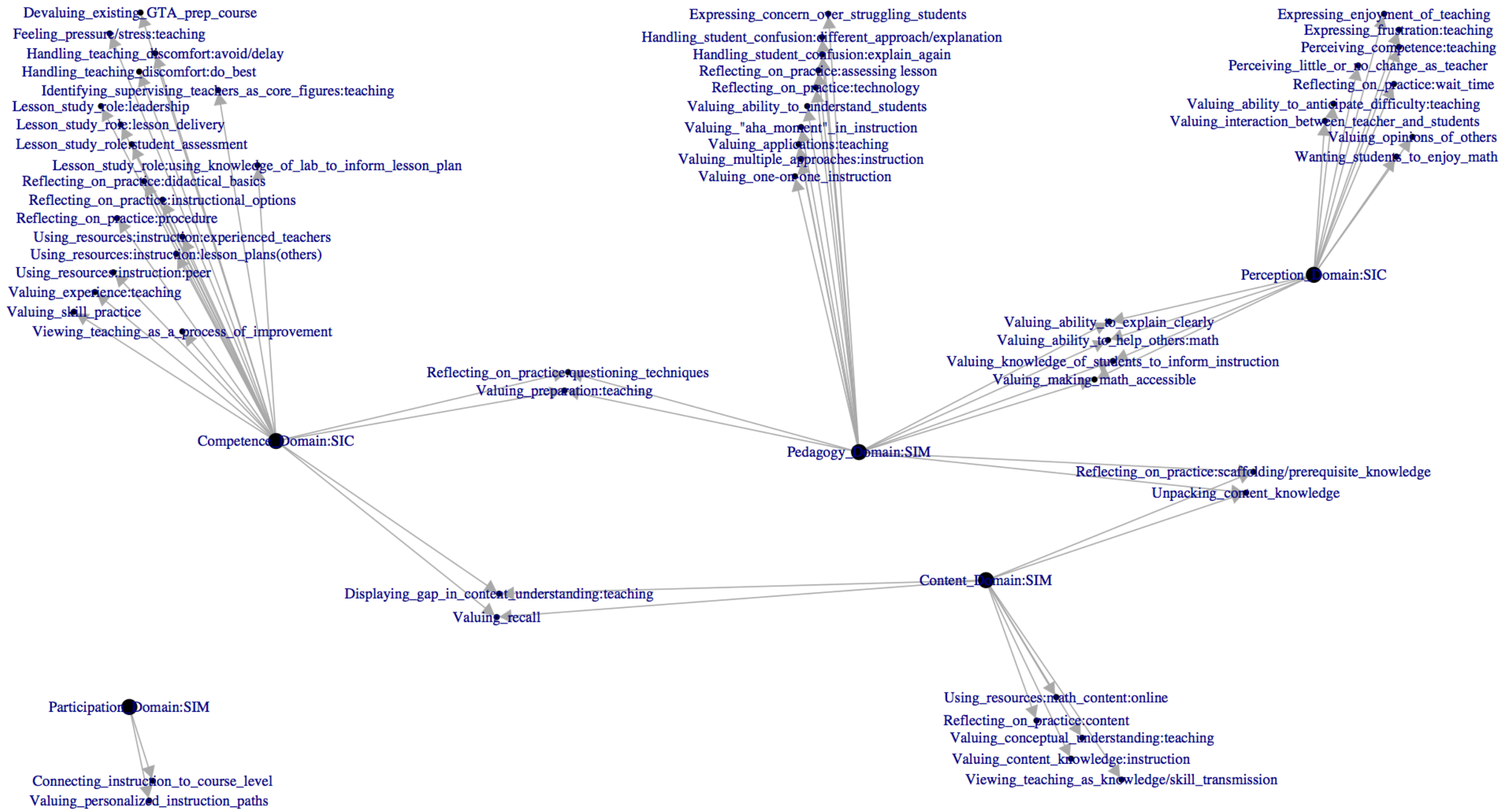


Figure 5.4.2: Dave's profile using codes associated with each of the five categories adapted from Van Zoest & Bohl's framework for teaching identity development, and incorporating data from both interviews, both lesson studies, and all six written reflection prompts. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the five code categories is displayed with a larger vertex size than the codes associated with that category.

The strongest benefit Dave seemed to be aware of drawing from his first semester experiences with teaching was reinforcement of previously held views:

> "[S]omething that was reaffirmed is that repeatedly going through a lesson is extremely helpful. In [precalculus], going over a topic for the first time, you are bound to make mistakes or forget to mention a small piece of information that may be helpful for a different variation of the problem ... [this] reaffirmed the importance of preparation, especially with practicing the lesson beforehand."

Nonetheless, despite a self-perception of no change in his views of teaching or of mathematics, the situated practice in the Precalculus classroom, together with the case discussions, appears to have had a similar impact on Dave's views of teaching and the nature of mathematics as it did on Cora's. He shifted his view on several key epistemic prompts. For example, he no longer believed that instruction on computational procedure must precede understanding (see Table 5.4.1). Those changes were consistent across the epistemic subscale and of the four, Dave exhibited the greatest shift towards a constructivist view of mathematics.

Dave also exhibited the greatest shift away from teacher identity, but the items contributing to that shift are largely related to views of the role of teaching in the professional identity of academic mathematicians. By the end of the first semester, he had come to believe that math professors are expected to spend most of their time on research and very little time thinking about teaching. He had also decided his math professors didn't seem to enjoy teaching. His own self-efficacy as a teacher, on the other hand, had improved, with Dave now disagreeing with the statement that no matter how hard he tried, there would be some math topics he couldn't teach well (see Table 5.4.1).

Consistent with his goals upon entering graduate school, the Professional Images (Teaching) category was very active in Dave's profile under Ronfeldt & Grossman's framework, while his Professional Images (Mathematician) category was relatively weak (see Figure 5.4.3). His primary images of mathematicians were related to valuing recall and breadth, and he equated being a mathematician with problem-solving. His images of teachers included both tasks associated with teaching, such as lesson planning and grading, and value statements that heavily informed his provisional selves.

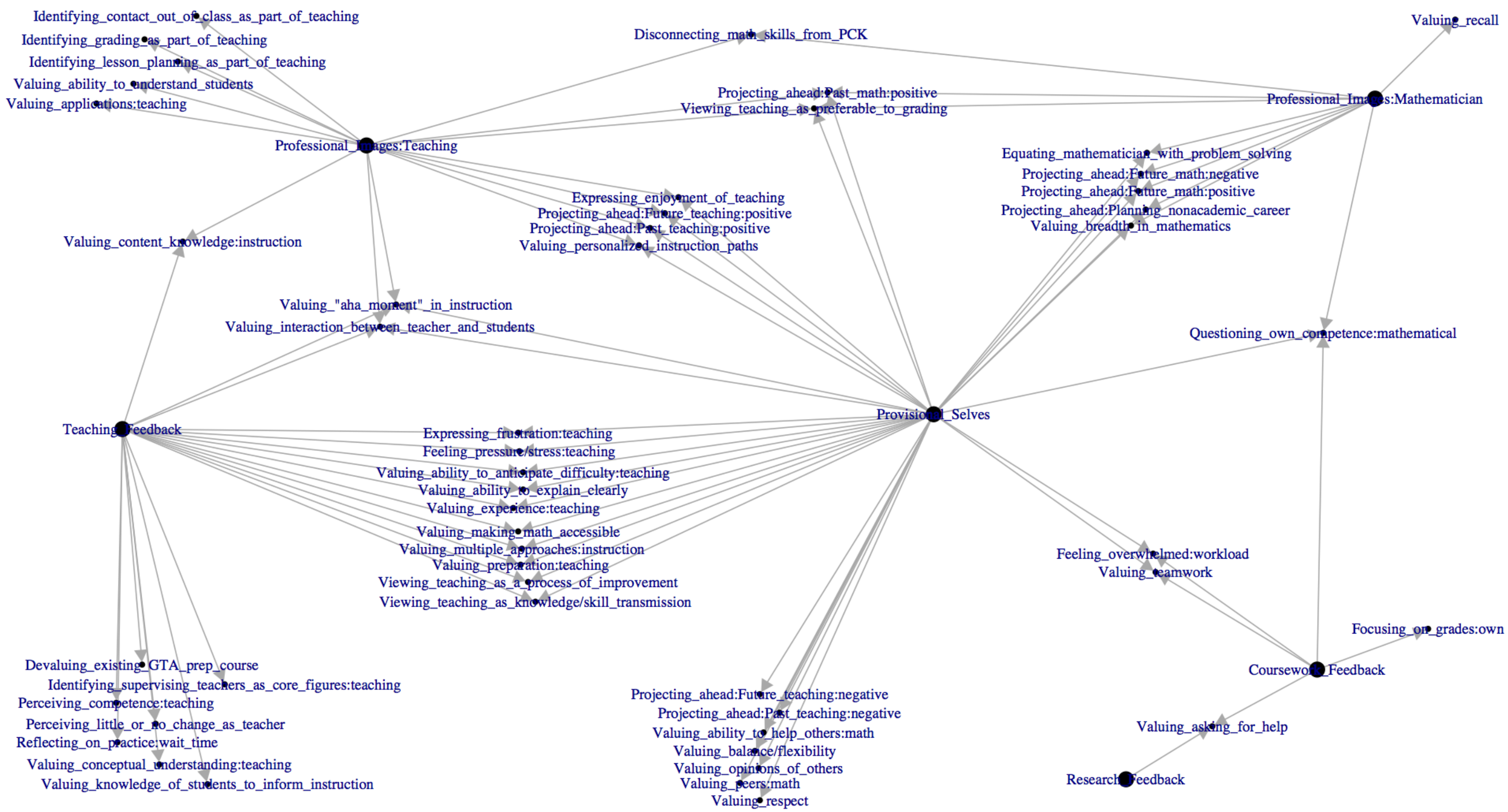


Figure 5.4.3: Dave's profile using codes associated with each of the six categories adapted from Ronfeldt & Grossman's framework for professional identity development, and incorporating data from both interviews, both lesson studies, and all six written reflection prompts. Location of vertices is for clearest display of labels and is not meant to imply any relative importance. Each of the six code categories is displayed with a larger vertex size than the codes associated with that category.

Like Bill and Cora, Dave felt overwhelmed and frustrated by his mathematical coursework and began to question his own mathematical competence. Dave, however, found support through interactions with his peers, and learned to value teamwork as a new facet of his provisional selves. He particularly saw a difference between his experiences during his provisional semester with an hourly grading assignment and his experiences on full teaching assistantship.

> "[T]he biggest difference I probably had was working with other grad students, whether it was assignments we had and just sitting down and working on them together and talking through them or studying together. Um, um, I mean ... And if I'm completely honest, I ... The ... The first semester, I didn't do well, and I tried to just do it by myself. And that first TA position I had was grading papers, so I just went to my mailbox, collected 'em, went home, graded 'em. Um, and then the ... In the fall of my second semester, I was TA'ing for [researcher's] course, and it introduced me to some other grad students, um, that I happened to have classes with. So. it was an easy avenue to say like, um, 'Hey, we have this homework, like, what did you get?' or 'Do you wanna work on it?' And, um, being able to bounce ideas back and forth, or maybe you do something, and then it's not quite right and you just went off on that track, um, they could say like, 'Hey, I-I think this is what you need to do.' So, it's nice to bounce ideas back and forth. Um, that was a big thing for me."

In the process of those interactions, Dave became more involved with peers who were planning non-academic careers, and they encouraged him to explore those options.

> "Um, and then someone said you should look into actuary science so it's something that I've kind of considered like I was saying, and being able to go back and teach later. And uh, meeting more people, more graduate students in the department kind of like I guess changed my thought process on being a teacher or being someone who more uses I guess you would say a mathematician- Someone who is using the math to solve some problem that they have um- ... I looked into it. It did seem pretty interesting. Um ... Just last night actually at intramural volleyball with the department uh, that guy in the department who's doing just a Master's program which is what I'm trying to do, and his concentration is statistics, and he's trying to do the actuary. He had just taken the first exam for it and so, this class prepared him really well for it. They offer the class here, so that was kind of reassuring, so that's the course I'm in now."

By the time of the second interview, Dave had solidified his decision to abandon teaching in favor of actuarial work.

> "Um, my current plan is to finish my Master's degree in December, and this summer I'm taking a few graduate courses, but also a few undergrad courses that are required for the ... Um, to be certified to be an actuary, basically. And then, um, because I have a full year lease at, uh, where I'm living, in the spring of next year I'll be also taking the remainder of the undergrad courses that I need to take as long with taking the ...

> The exams that go with being ... Becoming an actuary and being c ... Being certified. So, the plan is basically to graduate from Clemson with my Master's in December, and then in the spring complete all requirements for becoming an actuary, and then not the coming fall but the next fall, pursue a career in actuary work."

His full profile using the categories from all three frameworks, but only data from Interview 2, provides some insight into the factors influencing that decision (see Figure 5.4.4). Although he continued to express enjoyment of teaching, helping make math accessible to others, and interacting with students, most of his discussion of teaching was focused on the non-teaching associated tasks such as planning and grading. That is likely linked to the fact that he felt overwhelmed by his workload in graduate school and also felt some frustration and stress in teaching. Those reactions were situated within his experience balancing continued coursework with a solo teaching assignment for the first time.

> "Because I started in the spring of the previous year, uh, this semester was the first semester I was allowed to teach at Clemson, so I taught a course. And, um, I taught Monday, Wednesday, Friday for 50 minutes or a hour, whatever you wanna call it. And, um, even though I was only teaching for three hours of the week, you also have probably three hours of planning, plus you have any grading, plus you have answering emails, so it quickly adds up with the total amount of time you have for the teaching requirements. So, I would imagine with an actual teaching job, you don't just ... Once you leave the high school or whatever level you're teaching at, your work doesn't necessarily end. You have to continue grading, or planning, or answering emails. That's probably the biggest thing that's taken time, is emails from students about their grade, about meeting for extra help, about homework questions. I mean it ... It kinda never ends."

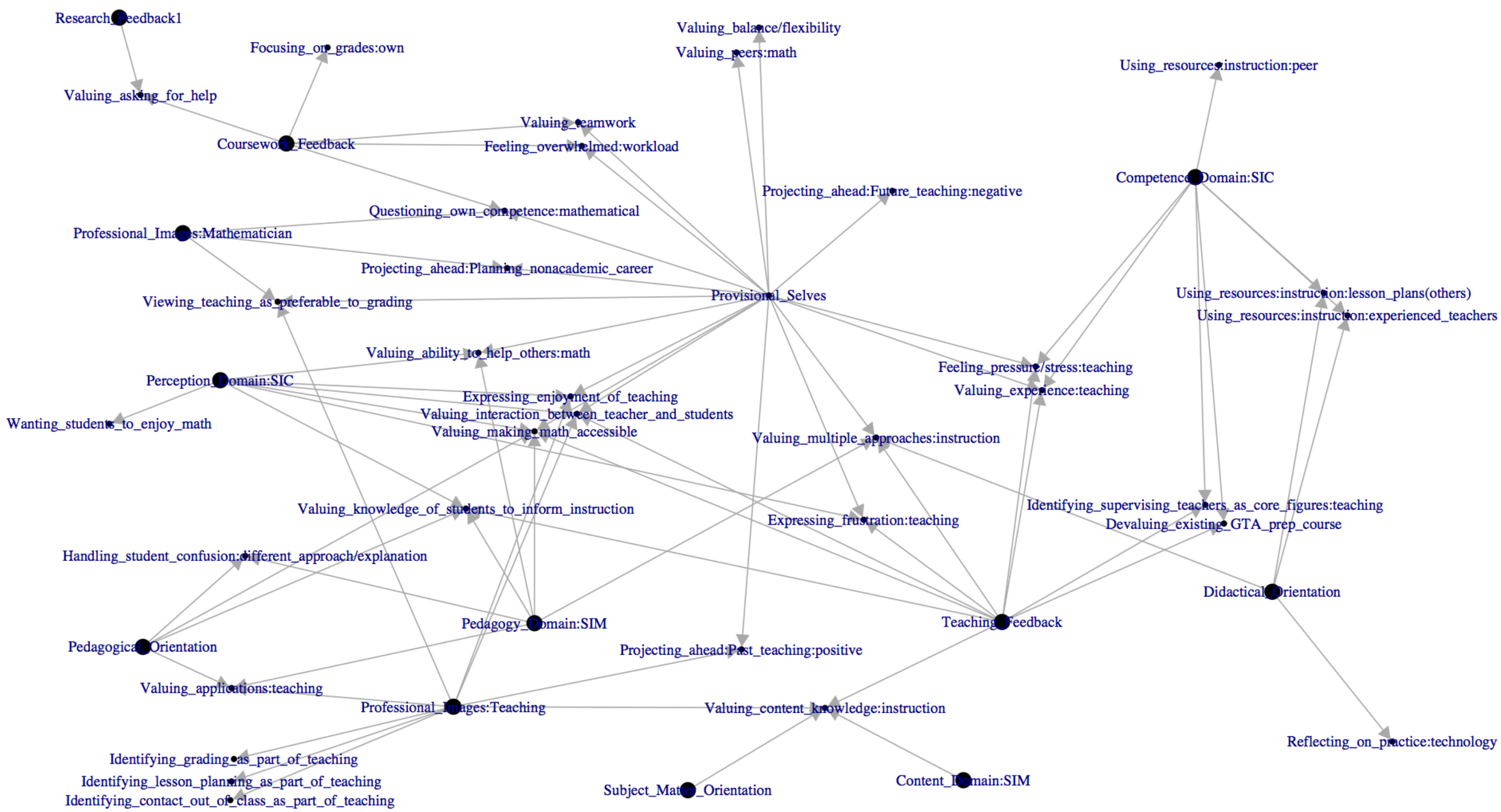


Figure 5.4.4: GTA4's Profile Using All Three Frameworks for Second Interview Data. Dave's profile using all codes applied to Interview 2, showing interrelation between the three theoretical frameworks. Each of the code categories is displayed with a larger vertex size than the codes associated with that category.

The code of valuing balance and flexibility appeared for the first time in Dave's provisional self at the time of the second interview, and it helps contextualize his decision to pursue an actuarial career rather than teaching.

> "I mean, if I could go wake up, and be in the job at nine and leave at five and not have to worry about anything job related, I would be completely fine with that, um, because, I mean ... I don't know. Growing up, I guess, that's kind of what I expected a job would be. Um, but kind of like what I said with an actual teaching job, where you have a lot more added on ... Even if you have all your planning done, you're a couple years into your teaching position, um, you still have the grading aspect. You still have the answering to students. You still have the ... Depending on what level you're at... A high school ... Uh, the parent interaction if you're in the high school level with checking in on their students or, um, answering to maybe the principal or whoever."

In keeping with his plan at the end of the first year of this study, Dave completed his Master's in December and obtained a job as an actuary. At that time, he had no plans to return to teaching at the secondary or post-secondary level.

Chapter 6

Cross-Case Analysis

Both the similarities across the four cases and the differences between the four cases give us valuable information in addressing our research questions. As one facet of analyzing those similarities and differences within the coded qualitative data, we look at individual codes within each code category. Codes that appear in three or four of the cases we view as a similarity across cases, while codes that appear in only one or two of the cases we view as differences among the cases. On occasion, we may refer to a code as being 'universal' or 'uniform' meaning that it was present in all four cases, or as 'unique' meaning that it appeared in only one case. Absence of the term 'universal' or 'uniform' in reference to a similar code does not imply that the code is present in only three cases, nor does absence of the term 'unique' in reference to a difference code imply that the code appeared in two cases.

As in Chapter 5, we again refer to the participants by their pseudonyms in this chapter.

6.1 Analysis Using Beijaard et al.'s Framework for Teacher Identity

From the case artifacts, we saw that after the first case discussion, all four participants retreated to a purely or primarily subject-matter expert role. With the exception of Anna, they then moved back to a more central location, with a balance of contributions from each of the three positions of expertise. In all four cases, however, the GTAs remained lodged firmly in the subject-matter expert corner of the triangle. Given the nature of the combined course, this is not entirely surprising. In each of the case discussions, they were the figures with the most mathematical authority, having completed an undergraduate degree in mathematics and begun graduate studies in mathematics. They also had the least pedagogical authority, as the other group members had completed considerable amounts of education coursework and were concurrently enrolled in a pedagogy of mathematics course. At the beginning of the semester, neither the GTAs nor the undergraduates had much didactical authority, not having planned or conducted class sessions. As the semester wore on and the GTAs gained experience delivering mini-lessons in the precalculus classes, their willingness to assume didactical authority in the group discussions shows up as a shift towards the didactical corner of the triangle.

The coded interview and reflection data echo the strong leaning toward subject-matter and didactical orientations in the identity trajectories from the case arcs, but provide deeper insight into those trajectories. All four of the participants have a preponderance of codes associated with the didactical and subject-matter orientations. Moreover, while didactical and subject-matter issues were each discussed in 35 of the 43 coded data sources, pedagogical issues were only present in 20 of those 43. Within the didactical orientation category, we see far more similarities than differences among the participants. They were heavily concerned with what might be termed 'didactical basics': effective boardwork and use of technology, pacing, speaking volume, and wait time in questioning. They wanted students to pay attention and valued the ability to explain clearly. While they valued having multiple instructional options and desired student engagement, they viewed that engagement taking the form of developing questioning techniques within a structured lesson plan revolving around content delivery in the form of lecture. That is, they valued teacher-student interaction, but student-student interaction was not mentioned at any time by any of the participants.

When faced with student confusion, their first response was to explain again more slowly. Through reflecting on practice over the course of the semester, they began to value the ability to scaffold instruction to fill in gaps in missing prerequisite knowledge and viewed that as one aspect of anticipating difficulty in teaching mathematical content. In planning for instruction, they relied both on experienced instructors and on their peers.

The differences we see within the didactical orientation category are mostly small. Bill and Dave were concerned about their ability to use technology effectively, but purely from an operational standpoint such as how to turn on the projector and scroll down in the SmartBoard software. Anna and Bill both valued patience as a didactical tool for having students answer short questions posed by the instructor as part of a lecture format. Bill and Cora mentioned using online resources to plan instruction.

More significantly, Cora defined a 'real' classroom as one in which the same instruction was delivered to all of the students at the same time. She also equated a student's inability to learn with an instructor's inability to explain clearly, although both of those views were expressed during the first interview and appeared to have undergone a shift by the time of the second interview. At the other end of the spectrum, Dave was very taken with the individualized learning paths and ability to focus only on the content with which students struggled, going so far as to say he would probably use it in his own classrooms in the future if he were teaching at that level.

Within the subject-matter orientation pole, the participants uniformly valued content knowledge and recall, and all of them focused on mathematical content when reflecting on practice. We also saw similarities in their view of teaching as transmission of procedural skill while also valuing conceptual understanding. The juxtaposition of those two, and the way in which those codes are phrased, gives insight into the fact that although they valued conceptual understanding, they did not necessarily see it as the primary goal of instruction in mathematics.

The differences between the participants within the subject-matter orientation codes give some interesting insights as well. Two of the participants, Bill and Cora, were very similar to each other while also exhibiting differences from the other two. They shared their fear of making a mathematical error in teaching, their use of course textbooks to develop instructional strategies, and their perception having developed competence in unpacking mathematical content through participation in the seminar and situated practice. They also reflected on conveying social knowledge such as adhering to notational conventions and mathematical terminology. In addition, Cora expressed her own enjoyment of procedural manipulations while also beginning to question the balance of skill transmission with explanation of why those procedures are valid.

Anna and Dave, on the other hand, displayed marked differences from each other, as well as from the Bill/Cora pair. Dave, who had completed a student teaching experience prior to graduate school, had the unique code of relying on subject matter experts to develop instructional strategies. While he also mentioned experienced teachers and online resources for the same purpose, his first step in developing instructional strategy was very tellingly to discuss the content with experts. In contrast, Anna indicated that in planning instruction, she would first focus on the big picture in order to identify the key aspects that needed to be addressed.

Within the pedagogical orientation pole, we uniformly saw value placed on understanding students, wanting to help students, making math accessible, and attempting a different approach or explanation when a student was confused. All four perceived value in the precalculus teaching experience, but for different reasons. Cora and Dave, who had participated in education coursework as undergraduates, valued the opportunity to refresh and deepen their content knowledge and also to better understand the needs of the students as individuals through one-on-one interaction. Anna and Bill, who had not had education coursework, valued the opportunity to practice didactical skill in a low-risk environment. Despite similarities in aspects of the lab structure valued by Cora and Dave, Dave viewed the lab as a good model for personalized instruction while Cora dismissed it as not a 'real class'.

All but Bill demonstrated continued movement toward the center of the identity triangle beyond the time frame of the teaching seminar, but Bill moved to an entirely didactical perspective during that same time. During the first semester experience, all four developed increased awareness of unpacking content knowledge, which lies at the center of the Beijaard identity triangle and is the connecting link to an analysis of teacher identity through the lens of pedagogical content knowledge using Ball's pie (see Figure 1.4.2). However, in no case was that understanding robust enough to warrant the higher level analysis, although we anticipate based on these results that we may find that framework appropriate for use with more experienced GTAs.

It is worth noting that the lesson study experiences primarily reinforced didactical orientation, while the situated practice appears to have initially reinforced subject-matter and didactical orientations, with pedagogical orientation only gaining strength after the participants perceived development of didactical competence. The case discussions in seminar appear from the identity trajectories to have primarily strengthened subject-matter and didactical orientations, but the change in epistemic beliefs can be most closely attributed to the case discussions, and those changes would seem to support a shift towards a more student-centered learning environment and towards pedagogical orientation. We posit that the combination of case discussions with situated practice contributed jointly to the development of pedagogical orientation and a corresponding movement towards the center of the Beijaard triangle.

6.2 Analysis Using Van Zoest & Bohl's Framework for Teaching Identity Development

As noted in Chapter 4, Beijaard et al.'s framework nestles within the Content and Pedagogy Domains (Self-in-Mind) from Van Zoest & Bohl's framework, so we focus now on the Participation Domain (Self-in-Mind) and Perception and Competence Domains (Self-in-Community) from Van Zoest & Bohl's framework. Those domains all deal with how the participant interacts with and within a teaching community of practice. In particular, we are interested in the codes from Figure 4.6.1 that are anchored within those domains but not within Beijaard's model.

The Participation Domain (Self-in-Mind) is far and away the weakest domain for all of the participants. As a whole, they have little awareness of larger communities of teaching practice. Cora, despite her education background, had no codings within the Participation Domain at all, and Bill had only the sense that the precalculus teaching experience was 'the best you could ask for.' Anna and Dave both had a roughly formed idea that perhaps the type and focus of instruction might appropriately differ based on the level of the course. Anna, in addition, articulated a general desire to adapt her didactical approach to match what students would experience in subsequent coursework. Dave engaged in speculation about how the precalculus instructional model could be adapted within the constraints of a high school setting. Other than these limited impressions, there was no sense of fitting into a teaching community of practice beyond what was established in the seminar.

Within the Perception Domain (Self-in-Community) we are concerned with perceptions of self, perceptions of others, and perceptions of others' perceptions, and we see some commonalities within each of these arenas. All of the participants expressed both frustration with teaching and enjoyment of teaching. The enjoyment of teaching came largely from the value they placed on interactions between teachers and students, and on helping others with mathematics. Their frustration stemmed from a mix of valuing the opinions of others, both students and supervisors, and of questioning their own competence for teaching.

Beyond those commonalities, we see a sharp distinction between those who came from a secondary mathematics education background and those who came from a mathematics background with no education coursework. Anna and Bill, with no education background, placed a strong value on observing other teachers and perceived growth in their own didactical and pedagogical competence. Cora and Dave, on the other hand, expressed a strong desire for their students to enjoy mathematics, and they perceived themselves as having undergone little to no change as teachers. Both of them spontaneously recounted specific interactions with students in classroom settings, whereas Anna and Bill spoke more in generalities unless specifically prompted for particulars.

The Competence Domain (Self-in-Community) contains codes related to developing competence, negotiating roles in teaching, and assessing outcomes of practice. Unlike the Perception Domain, the differences across cases within the Competence Domain did not split easily along education background lines. Cora and Dave no longer formed a similar pair, nor did Anna and Bill. Instead, we see mostly codes that are unique to a single participant, and they are not variations along a theme, but rather relatively unrelated. One participant valued problem-solving as one negotiated aspect of teaching. Another valued skill practice, and the remaining two had no parallel codes. Bill was very concerned with his team having had equitable roles in the lesson study cycles while none of the others addressed that issue. Bill and Cora both identified textbooks as valuable resources in planning instruction, and also mentioned using their knowledge of the students in the lab to inform the lesson study. Dave was dismissive of the existing one-unit GTA preparation course while the others did not mention it. Anna and Cora both mentioned peers and professors as core members of the teaching community of practice, while Bill and Dave did not.

As with the Perception Domain, we do also find some degree of consistency among the participants. Particularly in light of the individual differences we see, the consistencies provide valuable insight to address several of our research questions. The participants uniformly expressed a desire to avoid teaching a topic they didn't feel they couldn't teach well, looked to their peers as resources for planning instruction, and valued the opportunity to gain experience as teachers. On the whole, they saw themselves as leaders within the lesson plan cycles and viewed teaching as a process of improvement. They expressed a feeling of stress related to teaching and generally saw their supervising instructors as core figures within the teaching community of practice. One of their preferred resources for planning instruction was lesson plans developed by others whom they saw as more experienced.

6.3 Analysis Using Ronfeldt & Grossman's Framework for Professional Identity Development

The four participants held similar views of mathematics and mathematicians when they entered the program. All of them referenced some combination of problem-solving, logical thinking, generating knowledge, and connections within and applications of mathematics. However, while their views of what mathematics entailed were similar, their choice of mathematics as a field of study fell into two distinct camps. Bill, Cora, and Dave had come to mathematics in large part because they saw themselves doing well in a field that most people found difficult. Anna, in contrast, came to mathematics because she enjoyed attempting to solve open-ended problems. We did not anticipate this sharp distinction, but it appears to have played a major role in how these four participants internalized feedback in their first year.

While their views of mathematics were similar, they entered with very different backgrounds for teaching. One had no background in education or tutoring. One had tutoring experience but no education coursework. Two had undergraduate degrees in secondary mathematics education, but only one completed student teaching and held a credential. They also had very different career plans. Three of the four planned to obtain Ph.D.'s, with one of those planning a non-academic career, one planning an academic career for the purpose of conducting mathematical research, and one planning an academic career for the purpose of teaching. Two of the four did not see themselves as teachers while two strongly identified as such, but all four had good feelings towards, and looked forward to, teaching.

Their knowledge of mathematical content was similar, as were their epistemic beliefs on entering. They generally believed in an instructor-driven view of knowledge transmission, with demonstration of procedural technique necessarily preceding conceptual understanding. They had very little awareness of larger communities of teaching practice and did not even necessarily see themselves as part of the local community of teaching practice within the seminar setting. Two of them liked to do outside reading about teaching and two did not. They generally believed that math professors were expected to teach well and were not expected to spend most of their time on research.

As they began to engage with the local community of teaching practice in the seminar setting and Precalculus classrooms, all of them underwent significant shifts in their views of teaching and their own teaching identity, although only Anna acknowledged those changes at the end of the first semester. Critically, all four viewed teaching as a process of improvement. Although they felt pressure, frustration, and stress in teaching and questioned their own competence at times, they simultaneously valued the experience and expressed enjoyment in teaching. Although we did not specifically probe for differences in "mindset" [Dweck, 2006,Dweck and Leggett, 1988], there is direct evidence that all four had a 'growth mindset' for teaching: they viewed challenges as opportunities to improve rather than as evidence of a lack of innate ability that could not be overcome.

The manner in which they internalized course and research feedback stands in stark contrast to internalization of teaching feedback. Here again, they expressed frustration and stress and questioned their own competence. Bill, Cora, and Dave, however, entered with an arguably 'fixed mindset' towards mathematics. Unlike teaching, they viewed mathematics as something you were either good at or not. Part of what drew them to mathematics, in fact, was that they *were* good at it and it set them apart from their peers. All three of these participants responded to challenges in their graduate coursework with phrases such as, 'I just don't know if I personally can..." All three left graduate school after completing a Master's even though two of the three had intended to complete a Ph.D. Anna, on the other hand, used similar language for both coursework and teaching challenges; she saw both settings as opportunities to gain competence. She remains in the Ph.D. program.

This aspect of 'mindset' is not part of Ronfeldt & Grossman's model, but it appears to be a critical factor in interpreting how the first-year feedback affected provisional selves. Within the teaching feedback category, we see strong similarities among the four participants, all of whom appear to have had a similar mindset. The interplay with provisional selves, however, and particularly with projecting ahead to future images of self, was very different for the four participants. The role of teaching in provisional selves appears highly dependent on potential paths in and beyond the graduate program. Since graduate plans were strongly affected by coursework feedback for Bill, Cora, and Dave, we see their future projections of self either excluding teaching (Bill and Dave) or including teaching at a different level than originally planned (Cora). In Cora's case, she additionally expressed that if she could, she would choose to only teach advanced students. Their teacher identity was diminished, not because of teaching experiences and feedback, but rather because they internalized setbacks in their coursework as insurmountable and changed their career goals.

Chapter 7

Discussion and Conclusions

7.1 Limitations

There are certainly significant limitations to this study. The participants were all native-English speakers who had attended four-year colleges where the focus was on good instruction over research. The impact of seminar activities in professional preparation of non-native English speakers or of graduate students emerging from undergraduate programs at research-focused universities might be far different. More significantly, the construction of a teaching community of practice that included undergraduate preservice secondary teachers resulted in a very different first semester teaching experience than that experienced by the general GTA population. We must therefore be very cautious about making inferences to the general population. However, the insights we gained from these participants under these circumstances not only provide partial answers to our research questions, but also point clearly to next steps for extending the results.

The survey was not validated for this population, so the conclusions we can draw from test/retest with this group are limited. We have no additional data sources examining epistemic beliefs. The changes in that subscale are intriguing, and we cannot adequately situate them within our frameworks without additional context. The interview data identifies several areas that we wish had been reflected in the survey, including issues of motivation, recognition, and grit.

We found that we needed a better understanding of whom the participants saw as regular members of the community of practice. Although several classes of individuals were identified, we were not able to evaluate in a robust way the relative weight assigned to feedback from different members. It would be helpful to have a richer understanding of the range of interactions the participants had outside of the combined seminar. Future work will include collecting data about those interactions.

The cases were drawn from high school classrooms rather than college classrooms. Although the mathematical content was similar to that of the precalculus course, the case arcs did not allow for discussion of contextual issues relevant to higher education, nor did they address the mathematical content of calculus. For future implementations, we might seek to use cases based on college mathematics classrooms, rather than secondary mathematics classrooms.

The lesson study cycles were intended as a connection to a more advanced framework for analyzing teacher identity, but were beyond meaningful reach for the first-year GTAs. They hold promise for professional development with more advanced graduate student teachers, but with this population they did not promote the growth in pedagogical content knowledge that we wished to analyze, and proved wholly unsatisfactory for analysis under the more advanced lens.

Locating the participants in Beijaard et al.'s framework using relative proportions of codings within each category is only viable if each comment can reasonably be considered to reflect similar value. The case discussion prompts created such conditions, but the semi-structured interviews and open-ended reflective writings did not. Our sense of identity trajectory as a quantifiable construct is thus limited to one of our data sources.

The use of student performance data also has limitations. Among them, the grading structure of the course may have artificially inflated the second and third midterm averages and artificially depressed the final exam average. In addition, since most of the students were first-semester freshmen, we could not control for college GPA, nor did we have IRB approval to pull high school GPA or SAT/ACT scores for the students in those courses.

7.2 Addressing the Research Questions

Despite the limitations discussed above, we can provide meaningful insight into each of the research questions posed in Chapter 1.

7.2.1 What messages do first-year graduate students receive, and from whom, about the role of teaching in professional identity as a mathematician?

The participants' perceptions of the messages regarding teaching appeared to be related in part to the message they expected to receive. Three of the four entered graduate school expecting teaching to be valued, particularly by professors. Those three indicated that they received messages of one type or another from professors indicating that teaching was not valued, and all three concluded the year believing that professors spend little time thinking about teaching and most of their time doing research. Anna, in contrast, entered graduate school expected teaching to be of little importance. She indicated that she received messages that teaching was in fact valued, and concluded the year believing that professors spend time thinking about teaching, but also that they are expected to spend most of their time on research. All four received impressions about teaching that were in contradiction to expectations, and all four changed their views of how professors view teaching. These messages, together with the coursework and research feedback from other core members of the community, had a clear impact on the provisional selves of the participants.

From peers, the participants received mixed messages and recognized them as such. They indicated that although some of their peers valued teaching strongly, others spent as little time as possible thinking about it. Although three of the four participants recognized supervising teachers as sources of messages about teaching, it is unclear what impact those messages had on the participants. They certainly assigned value to the influence of those supervisors in helping them develop effective teaching practice, but it is less clear what impact those experiences had on provisional selves of the participants. It is here that we feel most keenly the lack of a rich understanding of interactions within the graduate community of practice. Given the mixed nature of those messages, it would be of great value to understand better the weight assigned to the message based on source.

7.2.2 What sources of information do graduate students rely on as they navigate expectations during the first year in graduate school?

The role of peers in meeting first-year expectations was clear. Participants turned to their peers far more than to any other source for help with coursework, developing instructional strategies, learning to balance demands, and even exploring career possibilities. Once again, we lament our superficial knowledge of this network. Participants interacted with their peers in all of the arenas that might reasonably be balanced as part of an overall professional identity, and valued those interactions for navigating the first-year experience. The impact of those interactions is not captured as fully as we would like within any of the three frameworks we considered. The peer-to-peer teaching interactions are captured to some extent within the Self-in-Community Domains of Van Zoest & Bohl's framework, but primarily for the interactions that occurred within the teaching seminar. Peer-to-peer teaching interactions outside of that context are under-represented. Peer-to-peer coursework and research interactions play a role in Ronfeldt & Grossman's framework, and it is within that framework that we would like to embed a richer understanding of the effect of the feedback loop between those interactions and provisional selves within the community. Understanding that social network is a critical next step in developing a robust model for teacher identity development within an overall professional identity for mathematics graduate students.

7.2.3 What aspects of teacher identity are reinforced or weakened by first-year graduate school experiences, and specifically by the enriched teaching experience?

This question is most appropriately answered within the context of Beijaard et al.'s framework. We found that at the start of the year, prior to taking on lead instructional roles, the GTAs were concerned with all of pedagogical, didactical, and subject-matter issues. Once they began to lead instructional activities, they became extremely, indeed almost exclusively, concerned with didactical issues. Subject-matter concerns took on less importance, and pedagogical concerns were virtually abandoned. However, as they gained experience, their attention to pedagogical issues grew and deepened. Not only is this promising within the context of the study, but it underscores the importance of promoting such growth in the first year, prior to GTAs assuming sole responsibility for courses without ongoing support to develop a balanced teacher identity.

7.2.4 What impact do future goals have on how graduate students balance first-year expectations in graduate school?

All of our data point to this question being backwards. That is, the future goals of the participants in this study were sufficiently malleable that first-year expectations influenced those goals far more heavily than vice versa. In fact, all four had significant changes in those goals, largely on the basis of first-year feedback. Dave planned to get a Master's and start teaching high school; he is now an actuary. Cora planned to get a Ph.D. and be a college professor with a focus on teaching; she stopped with a Master's and is now teaching high school. Bill planned to get a Ph.D. and be a college professor with a focus on research; he stopped with a Master's and is seeking an industry position. Anna is the closest to pursuing her original goals. She planned to get a Ph.D. and pursue an industry position; she is still pursuing a Ph.D. but now considering remaining in academics.

These results support previous research indicating that graduate school is a time of enormous change and fluid provisional selves. It also provides evidence that strong professional development and support in the first year of graduate school could have significant impact on the provisional selves that graduate students adopt as permanent aspects of their professional identity.

7.2.5 What impact does the enriched first-year teaching experience have on subsequent teaching practice?

Student performance in coordinated courses taught by the participants exceeded student performance in the same courses taught by graduate students with the same level of teaching experience and, in one case, matched that of experienced full-time lecturers teaching the same course. However, we note again that we only have reliable data for student performance for two of the four participants, and only for the first semester of teaching practice. While our limited data is promising, we hesitate to draw causative conclusions in such a limited context.

We note also that three of the four participants identified less closely with teaching at the end of the combined course with case study discussion. Those same three participants had a shift towards a more constructivist view of mathematics and knowledge. Their reflective writings indicate that these two shifts may be linked. They see themselves as less competent at teaching than they did at the start of the semester, and they place greater importance on understanding how students are thinking about mathematics as a central facet of being a good teacher. These shifts, then, may be considered as an overall positive within the context of helping graduate students develop effective teaching strategies and ways of understanding of students as learners.

7.2.6 How do mathematics graduate students experience the phenomenon of their first teaching experience in graduate school?

While we can only speak to the experiences of the participants who were included in an enriched teaching community of practice during the first semester, we can draw some limited conclusions towards this larger question. As an overall experience, the first year of mathematics graduate school is enormously stressful both in terms of workload and level of difficulty. We saw all four participants express that stress along multiple fronts, and question their own competence in multiple ways. The teaching experience within that larger picture is also stressful, but markedly less so than the coursework expectations. The participants all expressed enjoyment of teaching, valued the interactions related to teaching, and perceived growth in their own competence as teachers, even while questioning that competence at times. Much of the frustration they voiced about teaching centered around issues of wanting to be better able to help students, deliver effective instruction, and manage their time effectively. The low-risk teaching experience in the Precalculus lab setting was a critical positive factor for each of them, although in different ways.

7.3 Theoretical Implications

The results from this study contribute considerably to our understanding of how first-year experiences affect mathematics graduate students' professional identities, and in particular their teaching identities. We are left with a much clearer idea of which frameworks from research on related populations are most appropriately adapted for this population, how to adapt them, and what gaps in those frameworks need to be addressed for use with this population.

Beijaard et al.'s static model of teacher identity development is eminently suitable for this population. Our adaptation to a dynamic model of identity trajectory within the framework captured stages of development that were not readily apparent at a larger scale. We speculate that the rate of change for personal location within the framework will slow considerably as teacher identity stabilizes with experience, and that use of the dynamic adaptation will be most significant among first- and second-year graduate students in whom we see rapid change. Moreover, we anticipate that for graduate students whose identities stabilize near the center of the identity framework, it will be appropriate to explore next stages of teacher identity development within Ball, Thames, & Phelps' more advanced framework that proved unsuitable for the first-year participants.

Van Zoest & Bohl's framework proved largely suitable for a broader view of teacher identity development for this population, and indeed served to highlight some ways in which we might better support that development for graduate students. The graduate students interacted within their local and limited community of teaching practice in ways that were consistent with those of pre-service and student teachers at the secondary level. The exception is within the Participation Domain, where we see that graduate students have little awareness of broader communities of teaching practice within the mathematics community. Use of this framework also brought to light the fact that the graduate students saw the community of practice within the teaching seminar as time-limited, and themselves as a separate subgroup within the community. They interacted differently with the PSTs than with their graduate student peers. Nonetheless, the framework itself appears to be well-suited to examination of teacher identity development as a negotiated practice within the graduate student community. Beijaard et al.'s framework provides granular identity snapshots and individual identity trajectories; Van Zoest & Bohl's framework helps us understand the internal and external processes driving movement from one position to the next in the identity trajectory.

Ronfeldt & Grossman's framework contains critical components for examining the fluid nature of first-year graduate students' provisional selves and the factors affecting those selves. Of the three, however, it required the most adaptation for this population and cast into stark relief the areas we most need to explore further in order to develop a robust model for mathematics graduate student identity development. While the frameworks nest well to provide insight into teaching practice at various levels of granularity, we are still left with considerable questions about how the teaching experiences fit into the overall first-year experience. Our understanding of the interaction between teaching, research, and coursework contexts is not adequate to explain the effects we saw on provisional selves with respect to teaching identity. What is clear is that those communities have separate existence for the participants; there is not a single "graduate school community of practice" but rather separate communities within which the participants negotiated distinct identities and roles. We need a better understanding of how graduate students interact within course and research communities of practice, and how those communities overlap and influence one another in graduate school.

An emerging area of research in teacher identity development involves the idea of *productive friction*. Under the view that identity is negotiated through participation in a community of practice, it is at the boundaries between communities that identity is most easily changed, particularly when there is conflict between the behavioral norms of the communities [Ward et al., 2011]. For preservice secondary teachers, that conflict typically occurs during the transition from undergraduate coursework and student teaching to the first field placement as a practicing teachers. Courses on pedagogy, learning theory, and classroom management as experienced in the university setting may be in direct conflict with cultural norms of the school setting [Beauchamp and Thomas, 2011, Jarvis-Selinger et al., 2010]. Changing the community of practice within which identity was previously negotiated requires a new set of negotiations along the self-in-mind to self-in-community continuum from Van Zoest & Bohl's framework.

The idea of productive friction at the boundary has not been previously explored with respect to development of a teacher identity among graduate students, but it seems likely that it plays a key role, and perhaps one that runs counter to the existing research. What we appear to be seeing is a blend of *productive friction* solely within the localized teaching community of practice, but what might be termed *counterproductive friction* at the overlaps between teaching, coursework, and research communities of practice. Identifying the primary sources of conflict could have significant ramifications for mitigating that conflict and promoting strong teacher identity development for mathematics graduate students.

Another area of research that may also prove critical in understanding this population is that of "mindset" [Dweck, 2006, Dweck and Leggett, 1988]. The four participants received similar messages during their first year, but internalized them in distinct ways, as discussed in Section 6.3. Those differences in how feedback from different sources impacted provisional selves may well depend in part of individual mindset. Exploration of that theory seems warranted, particularly as it impacts the provisional self feedback cycle during the graduate school experience.

Finally, the change in epistemic beliefs is very intriguing. Those changes are not adequately reflected in any of the frameworks we used, nor were they addressed directly in the interviews or reflective writings. Adaptation and integration of the three frameworks will ultimately require both that we better understand the nuances of multiple – perhaps conflicting – communities of practice within graduate school, but also that we understand the interaction between teaching practice, coursework and research feedback, and epistemic beliefs. We anticipate that the appropriate location for epistemic beliefs will be within the Content Domain of Van Zoest & Bohl's framework, and that changes to those beliefs will be reflected both in Van Zoest & Bohl's framework, and within an improved understanding of graduate school communities of teaching-coursework-research practice.

7.4 Practical Implications

This study is intended as a first step in developing the foundational understandings to better prepare mathematics graduate students to incorporate robust teaching identities and practices within their professional identities. We did not design a study to test the effects of any single intervention, but rather to develop foundational knowledge for meaningful assessment of such interventions in the future. Nonetheless, the findings from this study suggest from practical implications even at this stage.

We implemented modifications of three best practices from K-12 teacher preparation. Of those, the situated practice stood out as highly effective. All of the participants commented not only on the value of specific aspects of that practice in their own right, but also in comparison to the first-year teaching experiences of their peers in other assignments. The evidence strongly supports the value of situated practice that provides first-year graduate students the opportunity to interact directly with small student groups on a regular basis, with ongoing support within a teaching community of practice. In developing professional development programs for first-year graduate students, we strongly recommend a model that incorporates such experiences as a central component.

The use of case discussions in the preparation of mathematics graduate student teachers is also promising. Cases allow for fruitful dialogue focused on each of subject-matter, pedagogical, and didactical expertise. They also provide a vehicle for examining teaching practice and student thinking in a meaningful context but without the immediate pressure or personal jeopardy of a current teaching assignment. Most promisingly, case discussions establish a forum for elucidating specific aspects of teaching expertise in order to accelerate first-year graduate students toward the more balanced teaching identity exhibited by experienced teachers.

It is unclear whether the pedagogical richness added to the conversations by the secondary mathematics education majors in the combined seminar was an added value for the GTA participants, or whether the GTAs would have been better served by participating in case discussions in which they felt more comfortable expressing and exploring their pedagogical views rather than deferring to those with greater perceived expertise. In future implementations, we might seek a more even balance between senior undergraduate mathematics majors and graduate students, or we might consider teaching a case-based professional development course exclusively for mathematics graduate students but supplemented by readings on mathematical pedagogy.

The lesson study experience was of minimal value at this stage. We suspect it might prove more fruitful for GTAs whose teaching identities have stabilized near a balance of orientations, and who are ready to deepen their pedagogical content knowledge. We recommend reconsidering lesson study as an optional professional development opportunity for interested graduate students with at least one full year of experience as a teacher of record.

Further theoretical development, including a better understanding of how mindset and non-teaching community interactions affect provisional selves, will undoubtedly lead to additional implications for practice.

Chapter 8

Future Work

Currently, no good model exists for development of a teacher identity within a larger mathematician identity among mathematics graduate students, and professional development and instructional support for mathematics graduate students varies widely from program to program. The results of this study contribute to the foundational knowledge of how these students experience teaching and begin to develop a teacher identity during the first year of graduate school. They also point to clear next steps for development of a robust framework for this particular population so that we can better evaluate instructional practices and develop institutional policies that support new graduate students as they enter their teaching practice.

From this work, we have the start of a more complete model of professional identity development for mathematics graduate students. Using our revised model as a starting framework, we will next need to explore more deeply the issue of roles within the community of practice, as they are perceived by graduate students at different stages in the program. Simultaneously, we need to explore more carefully the role of feedback in coursework and research settings, particularly as they interact with teaching feedback in trying on provisional roles. This study focused primarily on teaching feedback and, as a result, it appears that we missed deeper probing into the impact of non-teaching messages on teacher identity. Within that work, we recommend exploration of aspects related to motivation, recognition, and mindset, as those surfaced in unexpected places within this study and likely play a larger role than was initially anticipated.

Once additional information related to the graduate school experience allows the construction of a more robust model for this population, the next step is development of a reliable, validated identity instrument to allow us to measure change in identity over time. Specifically, we anticipate subscales aligning roughly to each of Beijaard et al.'s orientation categories as well as subscales regarding the nature of mathematics, the role of teaching within the professional mathematics community, and motivational aspects yet to be determined. Such an instrument would allow more quantitative and large-scale measurements of effectiveness of professional development materials for graduate students.

Once a valid, reliable instrument is in place for measuring professional identity including teaching as a component, we will be in a position to make solid claims about effectiveness of professional development programs for mathematics graduate students. A robust model and assessment instrument might well also translate to related STEM disciplines.

Ongoing results of this and future work stemming from this study will be presented at conferences associated with professional organizations such as Psychology of Mathematics Education, Association of Mathematics Teacher Educators, American Society of Engineering Education, American Mathematical Society, Mathematical Association of America, American Educational Research Association, and the International . Results will also be submitted to appropriate journals, such as *Journal for Research in Mathematics Education, International Journal of Education Research, Teaching and Teacher Education, Educational Studies in Mathematics*, and*Studies in Higher Education.*

Appendices

Appendix A Bracketing Prompts

- What are my beliefs about teaching mathematics?
- How did my graduate school experiences affect my view of teaching mathematics?
- What were key factors influencing/shaping my view of teaching mathematics at the college level?
- What are my beliefs about graduate programs in mathematics in general?
- What are my beliefs about the graduate program in mathematical sciences at Clemson University?
- What are my beliefs about the role of teaching in the professional duties of an academic mathematician?
- What factors shaped my views about the role of teaching in the professional duties of an academic mathematician?
- What are my epistemological beliefs?
- How do my beliefs about teaching and mathematics carry through into a classroom setting?
- What are my beliefs about effective instruction for precalculus in a college setting for STEM majors?
- What are my attitudes and beliefs about each of the four individual participants?

Appendix B Survey

The initial survey included here differs from the one administered at the end of the semester only in the prefatory paragraph. The order and phrasing of the prompts was unchanged between administrations.

MATH 4080/9700
Initial Survey

For a significant portion of this class, you will be working in teams of 4 or 5 to plan, deliver, analyze, and revise mathematics lessons. In order to make effective teams, I would like to have a better idea of your viewpoints about mathematics, about teaching, and about how students learn mathematics. Please complete this survey as candidly as possible. There are no “right” or “wrong” answers.

NAME: ______________________________ DATE: ______________

Please respond to the following statements by marking the response that most closely corresponds to ***your*** belief. Make sure to answer every question. Mark only one response for each question.

		Strongly Agree	Agree	Disagree	Strongly Disagree
1.	Mathematics is useful for the problems of everyday life.	o	o	o	o
2.	Mathematics is something I enjoy very much.	o	o	o	o
3.	I enjoy teaching mathematics to others.	o	o	o	o
4.	College math professors are expected to teach well.	o	o	o	o
5.	Time should be spent practicing computational procedures before students are expected to understand the procedures.	o	o	o	o
6.	It is important for a student to know how to follow directions in order to be a good problem solver.	o	o	o	o
7.	I like the easy mathematics problems the best.	o	o	o	o
8.	I don’t do very well in my mathematics courses.	o	o	o	o
9.	My math professors show little interest in the students.	o	o	o	o
10.	Working mathematics problems is fun.	o	o	o	o
11.	I feel tense when someone talks to me about teaching math.	o	o	o	o
12.	I look forward to teaching math.	o	o	o	o
13.	Students should understand computational procedures before they master them.	o	o	o	o
14.	Students should understand computational procedures before they spend much time practicing them.	o	o	o	o
15.	Teachers should teach exact procedures for solving mathematical problems.	o	o	o	o
16.	The instructional sequence of math topics should be determined	o	o	o	o

		Strongly Agree	Agree	Disagree	Strongly Disagree
	by the order in which students naturally acquire math concepts.				
17.	I feel at ease in a mathematics class.	o	o	o	o
18.	I like to do outside reading in mathematics.	o	o	o	o
19.	There is little need for mathematics in most jobs.	o	o	o	o
20.	Mathematics is easy for me.	o	o	o	o
21.	I feel at ease talking about teaching.	o	o	o	o
22.	I like to do outside reading about teaching.	o	o	o	o
23.	Students learn mathematics best from the teacher's demonstrations and explanations.	o	o	o	o
24.	When selecting the next topic to be taught, a significant consideration is what students already know.	o	o	o	o
25.	I spend as little time as possible thinking about teaching.	o	o	o	o
26.	The natural development of students' mathematical ideas must be considered in making instructional decisions.	o	o	o	o
27.	I get bored when people talk about different ways to teach.	o	o	o	o
28.	Math professors spend very little time thinking about teaching.	o	o	o	o
29.	When I hear the word "mathematics" I have a feeling of dislike.	o	o	o	o
30.	Most people should study some mathematics.	o	o	o	o
31.	To be successful in mathematics, a student must be a good listener.	o	o	o	o
32.	Teachers should allow students to figure out their own ways to solve math problems.	o	o	o	o
33.	I don't like anything about teaching.	o	o	o	o
34.	My mathematics professors don't seem to enjoy teaching.	o	o	o	o
35.	Students should be allowed to invent ways to solve math problems before the teacher demonstrates how to solve the problems.	o	o	o	o
36.	Teaching is of great importance to our country's future.	o	o	o	o
37.	It is important to teach well in order to get a good job.	o	o	o	o

		Strongly Agree	Agree	Disagree	Strongly Disagree
38.	The instructional scope and sequence of math topics should be determined by the formal organization of mathematics.	o	o	o	o
39.	Sometimes I read ahead in my mathematics texts.	o	o	o	o
40.	No matter how hard I try, there are some math topics I cannot teach well.	o	o	o	o
41.	It doesn't disturb me to teach a math class.	o	o	o	o
42.	Students learn math best by attending to the teacher's explanation of how to do the activity.	o	o	o	o
43.	I enjoy talking to other people about teaching.	o	o	o	o
44.	I would like to spend less time in school doing mathematics.	o	o	o	o
45.	I would like a job in which I don't have to teach.	o	o	o	o
46.	Mathematics is helpful in understanding today's world.	o	o	o	o
47.	Mathematics should be presented to students in such a way that they can discover relationships for themselves.	o	o	o	o
48.	I usually understand what we are talking about in math class.	o	o	o	o
49.	When selecting the next topic to be taught, one must consider the logical organization of mathematics.	o	o	o	o
50.	I don't like anything about mathematics.	o	o	o	o
51.	I am good at teaching.	o	o	o	o
52.	The only reason I am teaching is because I have to.	o	o	o	o
53.	Students can figure out ways to solve many math problems without formal instruction.	o	o	o	o
54.	I have a good feeling towards teaching.	o	o	o	o
55.	My mathematics professors make mathematics interesting.	o	o	o	o
56.	Math professors are expected to spend most of their time on research.	o	o	o	o
57.	Students should solve mathematical problems before they master computational procedures.	o	o	o	o

		Strongly Agree	Agree	Disagree	Strongly Disagree
58.	My mathematics teachers are willing to give us individual help.	o	o	o	o
59.	It is important to me to understand the work I do in math.	o	o	o	o
60.	When a student makes an error, it is important to me to understand why.	o	o	o	o
61.	I get a feeling of satisfaction from teaching.	o	o	o	o
62.	I have a good feeling towards mathematics.	o	o	o	o
63.	Children should understand the meaning of multiplication and division before they memorize basic math facts.	o	o	o	o
64.	Sometimes I work more mathematics problems than are assigned in class.	o	o	o	o
65.	It scares me to have to teach math.	o	o	o	o
66.	My mathematics teachers don’t like students to ask questions.	o	o	o	o
67.	Students should master computational procedures before they are expected to understand how those procedures work.	o	o	o	o
68.	No matter how hard I try, I cannot understand mathematics.	o	o	o	o
69.	It is important to me to teach well.	o	o	o	o
70.	I feel tense when someone talks to me about mathematics.	o	o	o	o
71.	My mathematics teachers present material in a clear way.	o	o	o	o
72.	I remember most of the things that happen when I teach.	o	o	o	o
73.	Students learn mathematics best by figuring out for themselves the ways to find answers to math problems.	o	o	o	o
74.	I have a real desire to learn mathematics.	o	o	o	o
75.	If I don’t get how to work a mathematics problem right away, I never get it.	o	o	o	o
76.	Working with students upsets me.	o	o	o	o
77.	The teacher should demonstrate how to solve math problems before students are allowed to solve problems.	o	o	o	o
78.	I often think, “I can’t do it,” when a mathematics problem seems	o	o	o	o

		Strongly Agree	Agree	Disagree	Strongly Disagree
	hard.				
79.	I would rather watch someone else teach than teach a topic myself.	O	O	O	O
80.	It is important to know mathematics in order to get a good job.	O	O	O	O
81.	It doesn't disturb me to work mathematics problems.	O	O	O	O
82.	I would be happy if I never taught math.	O	O	O	O
83.	I would like a job that doesn't use any mathematics.	O	O	O	O
84.	Students should be told to solve problems the way the teacher has taught them.	O	O	O	O
85.	My mathematics teachers know when we are having trouble with our work.	O	O	O	O
86.	I often think, "I can't do it," when a student doesn't understand what I am teaching.	O	O	O	O
87.	I enjoy talking to other people about mathematics.	O	O	O	O
88.	I am good at working mathematics problems.	O	O	O	O
89.	I have a real desire to teach.	O	O	O	O
90.	If a student doesn't understand what I taught them, s/he must not have paid attention.	O	O	O	O
91.	You can get along perfectly well in everyday life without mathematics.	O	O	O	O
92.	I remember most of the things I learn in mathematics.	O	O	O	O
93.	The best way to teach problem solving is to show students how to solve one kind of problem at a time.	O	O	O	O
94.	It makes me nervous to think about doing mathematics.	O	O	O	O
95.	I think of myself as a teacher.	O	O	O	O
96.	I would rather be given the right answer to a math problem than work it out myself.	O	O	O	O
97.	Children will not understand multiplication and division until they have mastered some basic math facts.	O	O	O	O

	Strongly Agree	Agree	Disagree	Strongly Disagree
98. Most of the ideas in mathematics aren't very useful.	o	o	o	o
99. I will be happy when I am done taking math classes.	o	o	o	o
100. It would make me sad if I never got to teach math.	o	o	o	o
101. I think of myself as a mathematician.	o	o	o	o

Appendix C Written Reflection Prompts

Written Reflection 1 was administered in class at the conclusion of the first lesson study cycle. Each prompt was on a separate sheet, prefaced by this statement:

> Please take your time in responding individually to this prompt. It is intentionally open-ended and responses could be taken different directions. Your response should reflect your own experiences and thinking, so please don't discuss the prompt or your response with your classmates until after you have all completed all four prompts. If you need more room, feel free to continue on the back and/or attach additional sheets.

1. Describe your response if someone asked you to teach a topic that you did not feel you could teach well.
2. Describe your response when a student doesn't understand what you are teaching.
3. Describe your response when you don't understand what is being taught in a math class.
4. Describe your role in the first lesson study cycle.

Written Reflection 2 was administered in class on the last day of the semester. Each prompt was on a separate sheet, prefaced by this statement:

> Think back on your teaching experiences JUST THIS SEMESTER (whether in MATH 1050, MATH 1990, or your cooperating teacher's classroom).

1. Identify one teaching revelation/insight/surprise you had. Describe the setting, what happened, and how it will affect your teaching in the future.
2. Identify one mathematical revelation/insight/surprise you had. Describe the setting, the mathematics and your new understanding, and how it will affect your teaching of mathematics in the future

Appendix D First Interview Protocol

These are the eight initial prompts for the loosely structured first-round interview with the four graduate participants.

- Describe the first time you thought you might like to teach mathematics.
- Describe the first time you thought you might like to be a mathematician.
- What do you think is the most important skill to develop in order to be a successful mathematics teacher? Why is that skill important?
- What do you think is the most important skill to develop in order to be a successful mathematician? Why is that skill important?
- What do you need to know in order to teach solving inequalities?
- How has your view of teaching mathematics changed and/or evolved over the course of this semester?
- Describe a situation in which you felt confident teaching a mathematical concept.
- Describe a situation in which you did not feel confident teaching a mathematical concept.

Appendix E   Second Interview Protocol

This is one sample of an interview protocol. The prompts were similar in each case, but individualized based on the previous interview.

- When we spoke back in October, we talked about some of your experiences with doing mathematics research and with teaching mathematics. Could you describe for us what you foresee as a typical week in your professional life after you graduate?
- How is that "typical" week you just described different from an ideal week in your professional life after you graduate?
- What messages have you gotten in your first year of graduate school about the role of teaching in the professional life of a mathematician?
- What are the sources of those messages?
- What were the most important events in your growth as a mathematics graduate student this year? (Prompt for teaching-specific growth if necessary after initial response given.)
- When we talked in October, you identified knowing the material really well and not losing sight of where the students are as the most critical skills to develop in order to be successful as a math teacher. Has that view changed?
- What caused that change/solidified that view? (As appropriate based on response to previous prompt.)
- Are there any other skills you would now identify as important that you didn't before?
- Are there any other ways in which your view of teaching mathematics has changed and/or evolved over the course of this semester?

Appendix F Summary Table of all Codes Assigned to Catogories

Code	Beijaard et al.	Van Zoest & Bohl	Ronfeldt & Grossman
Acknowledging family influence to attend grad school			
Addressing research expectations			Research Feedback
Attempting to change student attitude	Pedagogical	Pedagogy (SIM)	
Connecting instruction to course level		Participation (SIM)	
Connecting to larger curriculum		Participation (SIM)	
Devaluing brute-force/guess-and-check	Subject-Matter	Content (SIM)	
Devaluing existing GTA prep course		Competence (SIC)	Teaching Feedback
Developing instructional strategies: working examples	Subject-Matter & Pedagogical	Content (SIM)	
Disconnecting math skills from PCK			Professional Images (Math) & Professional Images (Teach)
Disliking research			Research Feedback & Provisional Selves
Disliking theory			Coursework Feedback & Provisional Selves
Displaying gap in content understanding: teaching	Subject-Matter	Content (SIM) & Competence (SIC)	
Distancing self from students	Pedagogical (Negative)	Self-in-Community (Perception)	

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Table F.1 – continued from previous page

Code	**Beijaard et al.**	**Van Zoest & Bohl**	**Ronfeldt & Grossman**
Enjoying problem solving			Provisional Selves
Enjoying procedural manipulations	Subject-Matter	Content (SIM)	
Equating "real classroom" with delivering same instruction to all students at same time	Didactical	Pedagogy (SIM)	Professional Images (Teach)
Equating big school with devaluing teaching		Perception (SIC)	Professional Images (Math) & Professional Images (Teach)
Equating confidence with excitement		Perception (SIC)	
Equating inability to explain with student inability to learn	Didactical & Pedagogical	Pedagogy (SIM) & Perception (SIC)	
Equating lack of recall with dislike of topic: teaching	Subject-Matter	Content (SIM)	
Equating math success with natural gift			Professional Images (Math)
Equating mathematician with generating knowledge			Professional Images (Math)
Equating mathematician with problem solving			Professional Images (Math) & Provisional Selves
Equating teacher competence with student mastery/retention	Subject-Matter	Perception (SIC)	Professional Images (Teach) & Provisional Selves
Equating teaching with content delivery	Didactical & Subject-Matter	Pedagogy (SIM)	Professional Images (Teach)
Excluding supervising teachers as core figures: implicit		Perception (SIC)	Teaching Feedback

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Table F.1 – continued from previous page

Code	**Beijaard et al.**	**Van Zoest & Bohl**	**Ronfeldt & Grossman**
Expressing "calling" to be teacher			Provisional Selves
Expressing "calling" to get Masters			Provisional Selves
Expressing concern over struggling students	Didactical & Pedagogical	Pedagogy (SIM)	
Expressing confidence in content knowledge: teaching	Subject-Matter & Didactical	Content (SIM) & Pedagogy (SIM)	Teaching Feedback
Expressing enjoyment/love of math			Provisional Selves
Expressing enjoyment of teaching		Perception (SIC)	Professional Images (Teach) & Provisional Selves
Expressing fear of public speaking	Didactical	Perception (SIC)	Teaching Feedback
Expressing frustration: teaching		Perception (SIC)	Teaching Feedback & Provisional Selves
Fearing being seen as incompetent		Perception (SIC)	Teaching Feedback & Provisional Selves
Fearing making mathematical error: teaching	Subject-Matter	Content (SIM) & Perception (SIC)	Teaching Feedback
Feeling frustration: math coursework			Coursework Feedback & Provisional Selves
Feeling overwhelmed: workload			Coursework Feedback & Provisional Selves
Feeling pressure/stress: teaching		Competence (SIC)	Teaching Feedback & Provisional Selves
Focusing on grades: own			Coursework Feedback
Focusing on individual learner needs	Pedagogical	Pedagogy (SIM)	
Handling student confusion: assume student fault		Pedagogy (SIM)	

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Table F.1 – continued from previous page			
Code	**Beijaard et al.**	**Van Zoest & Bohl**	**Ronfeldt & Grossman**
Handling student confusion: assume teacher fault		Pedagogy (SIM)	
Handling student confusion: different approach/explanation	Pedagogical	Pedagogy (SIM)	
Handling student confusion: explain again	Didactical	Pedagogy (SIM)	
Handling student confusion: focus on student affect	Pedagogical	Pedagogy (SIM)	
Handling student confusion: remain positive		Pedagogy (SIM)	
Handling student confusion: remind process/rule	Subject-Matter	Content (SIM)	
Handling teaching discomfort: avoid/delay		Competence (SIC)	
Handling teaching discomfort: doing best		Competence (SIC)	
Handling teaching discomfort: relying on prepared notes	Didactical	Content (SIM)	
Handling teaching discomfort: thinking about big picture	Subject-Matter	Content (SIM)	
Identifying contact out of class as part of teaching			Professional Images (Teach)
Identifying gaps in preparation to teach		Pedagogy (SIM)	
Identifying grading as part of teaching			Professional Images (Teach)
Identifying lesson planning as part of teaching			Professional Images (Teach)

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Table F.1 – continued from previous page			
Code	**Beijaard et al.**	**Van Zoest & Bohl**	**Ronfeldt & Grossman**
Identifying peers as core figures: teaching		Competence (SIC)	
Identifying professors as core figures: teaching		Competence (SIC)	Teaching Feedback
Identifying supervising teachers as core figures: teaching		Competence (SIC)	Teaching Feedback
Lesson study role: content	Subject-Matter	Competence (SIC)	
Lesson study role: equitable tasks		Competence (SIC)	
Lesson study role: leadership		Competence (SIC)	
Lesson study role: lesson delivery	Didactical	Competence (SIC)	
Lesson study role: student assessment	Didactical	Competence (SIC)	
Lesson study role: using knowledge of lab to inform lesson plan	Didactical	Competence (SIC)	
Lesson study role: using knowledge of students to inform lesson plan	Pedagogical	Competence (SIC)	
Perceiving competence: teaching		Perception (SIC)	Teaching Feedback
Perceiving content gap in ALEKS	Subject-Matter	Content (SIM)	
Perceiving disconnect between peer/faculty: teaching attitude		Perception (SIC)	Teaching Feedback
Perceiving growth in competence: didactical	Didactical	Pedagogy (SIM) & Perception (SIC)	Teaching Feedback
Perceiving growth in competence: pedagogical	Pedagogical	Pedagogy (SIM) & Perception (SIC)	Teaching Feedback

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Table F.1 – continued from previous page

Code	Beijaard et al.	Van Zoest & Bohl	Ronfeldt & Grossman
Perceiving growth in competence: unpacking math	Subject-Matter	Content (SIM) & Perception (SIC)	Teaching Feedback
Perceiving little or no change as teacher		Perception (SIC)	Teaching Feedback
Perceiving others' perceptions: incompetence	Subject-Matter	Perception (SIC)	Teaching Feedback
Perceiving others' perceptions: weirdness		Perception (SIC)	Teaching Feedback
Projecting ahead: Future math: ambivalence			Professional Images (Math) & Provisional Selves
Projecting ahead: Future math: negative			Professional Images (Math) & Provisional Selves
Projecting ahead: Future math: positive			Professional Images (Math) & Provisional Selves
Projecting ahead: Future teaching: ambivalence			Provisional Selves
Projecting ahead: Future teaching: negative			Provisional Selves
Projecting ahead: Future teaching: positive			Professional Images (Teach) & Provisional Selves
Projecting ahead: Past math: ambivalence			Provisional Selves
Projecting ahead: Past math: negative			Professional Images (Math) & Provisional Selves
Projecting ahead: Past math: positive			Professional Images (Math) & Professional Images (Teach) & Provisional Selves
Projecting ahead: Past teaching: negative			Provisional Selves

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Table F.1 – continued from previous page

Code	Beijaard et al.	Van Zoest & Bohl	Ronfeldt & Grossman
Projecting ahead: Past teaching: positive			Professional Images (Teach) & Provisional Selves
Projecting ahead: Planning academic career			Professional Images (Math) & Professional Images (Teach) & Provisional Selves
Projecting ahead: Planning nonacademic career			Professional Images (Math) & Provisional Selves
Questioning balance of why and how in teaching math	Subject-Matter	Content (SIM)	
Questioning nature of mathematics			Professional Images (Math) & Provisional Selves
Questioning own competence: mathematical			Coursework Feedback & Professional Images (Math) & Provisional Selves
Questioning own competence: teaching		Perception (SIC)	Teaching Feedback & Provisional Selves
Receiving message from peers: balance		Competence (SIC)	Coursework Feedback & Teaching Feedback & Research Feedback
Receiving message from professors: communication			Coursework Feedback & Research Feedback
Receiving message from professors: de-emphasize teaching		Perception (SIC)	Teaching Feedback & Research Feedback
Receiving message from professors: know content			Teaching Feedback
Receiving message from professors: value teaching			Teaching Feedback & Professional Images (Math)
Reflecting on practice: assessing lesson	Didactical	Pedagogy (SIM)	

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Table F.1 – continued from previous page

Code	**Beijaard et al.**	**Van Zoest & Bohl**	**Ronfeldt & Grossman**
Reflecting on practice: assessing student understanding	Didactical	Pedagogy (SIM)	
Reflecting on practice: content	Subject-Matter	Content (SIM)	
Reflecting on practice: conveying social knowledge	Subject-Matter	Content (SIM)	
Reflecting on practice: didactical basics	Didactical	Competence (SIC)	
Reflecting on practice: instructional options	Didactical	Competence (SIC)	
Reflecting on practice: pacing	Didactical	Competence (SIC)	
Reflecting on practice: procedure	Subject-Matter	Competence (SIC)	
Reflecting on practice: questioning techniques	Didactical	Pedagogy (SIM) & Competence (SIC)	
Reflecting on practice: scaffolding/prerequisite knowledge	Subject-Matter & Didactical	Content (SIM) & Pedagogy (SIM)	
Reflecting on practice: student engagement	Didactical	Competence (SIC)	
Reflecting on practice: technology	Didactical	Pedagogy (SIM)	
Reflecting on practice: wait time	Didactical	Perception (SIC)	Teaching Feedback
Revoicing student attitude: negative		Pedagogy (SIM)	
Seeking opportunities to demonstrate competence			
Unpacking content knowledge	Subject-Matter & Pedagogical & Didactical	Content (SIM) & Pedagogy (SIM)	

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Table F.1 – continued from previous page			
Code	**Beijaard et al.**	**Van Zoest & Bohl**	**Ronfeldt & Grossman**
Using resources: instruction: content experts	Subject-Matter		
Using resources: instruction: experienced teachers	Didactical	Competence (SIC)	
Using resources: instruction: lesson plans(others)	Didactical	Competence (SIC)	
Using resources: instruction: online	Didactical	Competence (SIC)	
Using resources: instruction: peers		Competence (SIC)	
Using resources: instruction: textbooks	Subject-Matter	Competence (SIC)	
Using resources: math content: instructor	Subject-Matter		
Using resources: math content: online	Subject-Matter	Content (SIM)	
Valuing "aha moment" in instruction	Pedagogical	Pedagogy (SIM)	Teaching Feedback & Professional Images (Teach) & Provisional Selves
Valuing ability to anticipate difficulty: teaching	Didactical	Perception (SIC)	Teaching Feedback & Provisional Selves
Valuing ability to explain clearly	Didactical	Pedagogy (SIM) & Perception (SIC)	Teaching Feedback & Provisional Selves
Valuing ability to help others: math		Pedagogy (SIM) & Perception (SIC)	Provisional Selves
Valuing ability to understand students	Pedagogical	Pedagogy (SIM)	Professional Images (Teach)
Valuing applications: math			Professional Images (Math) & Provisional Selves
Valuing applications: teaching	Pedagogical	Pedagogy (SIM)	Professional Images (Teach)

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Table F.1 – continued from previous page

Code	Beijaard et al.	Van Zoest & Bohl	Ronfeldt & Grossman
Valuing asking for help			Coursework Feedback & Research Feedback
Valuing balance/flexibility			Provisional Selves
Valuing breadth in mathematics			Professional Images (Math) & Provisional Selves
Valuing challenge			Professional Images (Math) & Provisional Selves
Valuing communication: math			Professional Images (Math)
Valuing communication: teaching		Pedagogy (SIM)	Professional Images (Teach)
Valuing competence: math			Professional Images (Math)
Valuing competence: teaching		Perception (SIC)	Teaching Feedback
Valuing conceptual understanding: teaching	Subject Matter	Content (SIM)	Teaching Feedback
Valuing confidence: teaching		Perception (SIC)	Teaching Feedback
Valuing connections: math			Coursework Feedback & Research Feedback & Provisional Selves
Valuing connections: teaching	Subject Matter	Content (SIM)	Teaching Feedback & Provisional Selves
Valuing consistency in grading			Teaching Feedback
Valuing content knowledge: instruction	Subject Matter	Content (SIM)	Professional Images (Teach) & Teaching Feedback
Valuing creation of knowledge			Professional Images (Math) & Provisional Selves
Valuing didactical skill in teaching	Didactical	Pedagogy (SIM)	Teaching Feedback
Valuing enjoyment			

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Table F.1 – continued from previous page			
Code	**Beijaard et al.**	**Van Zoest & Bohl**	**Ronfeldt & Grossman**
Valuing experience: teaching		Competence (SIC)	Teaching Feedback & Provisional Selves
Valuing interaction between teacher and students		Perception (SIC)	Teaching Feedback & Professional Images (Teach) & Provisional Selves
Valuing knowledge of students to inform instruction	Pedagogical	Pedagogy (SIM) & Perception (SIC)	Teaching Feedback
Valuing logical thought			Professional Images (Math)
Valuing low risk environment for early teaching experience		Participation (SIM) & Competence (SIC)	Teaching Feedback & Provisional Selves
Valuing making math accessible	Pedagogical	Pedagogy (SIM) & Perception (SIC)	Teaching Feedback & Provisional Selves
Valuing multiple approaches: instruction	Didactical	Pedagogy (SIM)	Teaching Feedback & Provisional Selves
Valuing multiple approaches: math			Coursework Feedback & Research Feedback
Valuing observing other teachers		Perception (SIC)	Teaching Feedback & Provisional Selves
Valuing one-on-one instruction	Pedagogical	Pedagogy (SIM)	
Valuing open ended problems			Research Feedback & Professional Images (Math)
Valuing opinions of others		Perception (SIC)	Provisional Selves
Valuing patience	Didactical	Pedagogy (SIM)	Teaching Feedback & Provisional Selves
Valuing peers: math			Provisional Selves
Valuing personalized instruction paths	Didactical	Participation (SIM)	Professional Images (Teach) & Provisional Selves

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Table F.1 – continued from previous page			
Code	**Beijaard et al.**	**Van Zoest & Bohl**	**Ronfeldt & Grossman**
Valuing positive teacher attitude		Perception (SIC)	Professional Images (Teach) & Provisional Selves
Valuing preparation: teaching	Didactical	Pedagogy (SIM) & Competence (SIC)	Teaching Feedback & Provisional Selves
Valuing problem-solving			Professional Images (Math) & Provisional Selves
Valuing recall	Subject Matter	Content (SIM) & Competence (SIC)	Professional Images (Math)
Valuing respect/obedience			Provisional Selves
Valuing skill practice	Didactical	Competence (SIC)	
Valuing spiritual/religious			Provisional Selves
Valuing teamwork			Coursework Feedback & Provisional Selves
Viewing mathematics as difficult			Coursework Feedback & Provisional Selves
Viewing mathematics as lots of work			Coursework Feedback & Provisional Selves
Viewing mathematics as more than teaching			Research Feedback & Provisional Selves
Viewing mathematics as superior to other fields			Professional Images (Math)
Viewing teaching as a process of improvement		Competence (SIC)	Teaching Feedback & Provisional Selves
Viewing teaching as knowledge/skill transmission	Subject Matter	Content (SIM)	Teaching Feedback & Provisional Selves
Viewing teaching as part of being academic mathematician			Professional Image (Math) & Professional Image (Teach) & Provisional Selves

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Table F.1 – continued from previous page

Code	Beijaard et al.	Van Zoest & Bohl	Ronfeldt & Grossman
Viewing teaching as preferable to grading			Professional Image (Math) & Professional Image (Teach) & Provisional Selves
Wanting students to enjoy math		Perception (SIC)	
Wanting students to pay attention	Didactical	Perception (SIC)	
Wanting to emulate a role model: teaching			Professional Images (Teach)
Wanting to teach advanced students			Provisional Selves

Table F.1: Table of Codes Assigned to Categories within Each Framework.

Appendix G Code Categories Adapted from Beijaard et al.'s Model

Code	Didactical	Pedagogical	Subject-Matter
Attempting to change student attitude		✓	
Devaluing brute-force/guess-and-check			✓
Developing instructional strategies: working examples		✓	✓
Displaying gap in content understanding: teaching			✓
Distancing self from students		✓ (negative)	
Enjoying procedural manipulations			✓
Equating "real classroom" with delivering same instruction to all students at same time	✓		
Equating inability to explain with student inability to learn	✓	✓	
Equating lack of recall with dislike of topic: teaching			✓
Equating teacher competence with student mastery/retention			✓
Equating teaching with content delivery	✓		✓
Expressing concern over struggling students	✓	✓	
Expressing confidence in content knowledge: teaching	✓		✓
Expressing fear of public speaking	✓		
Fearing making mathematical error: teaching			✓
Focusing on individual learner needs		✓	
Handling student confusion: different approach/explanation		✓	
Handling student confusion: explain again	✓		
Handling student confusion: focus on student affect		✓	
Handling student confusion: remind process/rule			✓
Handling teaching discomfort: relying on prepared notes	✓		
Handling teaching discomfort: thinking about big picture			✓
Lesson study role: content			✓
Lesson study role: lesson delivery	✓		
Lesson study role: student assessment	✓		
Lesson study role: using knowledge of lab to inform lesson plan	✓		

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Table G.1 – continued from previous page			
Code	**Didactical**	**Pedagogical**	**Subject-Matter**
Lesson study role: using knowledge of students to inform lesson plan		✓	
Perceiving content gap in ALEKS			✓
Perceiving growth in competence: didactical	✓		
Perceiving growth in competence: pedagogical		✓	
Perceiving growth in competence: unpacking math			✓
Perceiving others' perceptions: incompetence			✓
Questioning balance of why and how in teaching math			✓
Reflecting on practice: assessing lesson	✓		
Reflecting on practice: assessing student understanding	✓		
Reflecting on practice: content			✓
Reflecting on practice: conveying social knowledge			✓
Reflecting on practice: didactical basics	✓		
Reflecting on practice: instructional options	✓		
Reflecting on practice: pacing	✓		
Reflecting on practice: procedure			✓
Reflecting on practice: questioning techniques	✓		
Reflecting on practice: scaffolding/prerequisite knowledge	✓		✓
Reflecting on practice: student engagement	✓		
Reflecting on practice: technology	✓		
Reflecting on practice: wait time	✓		
Unpacking content knowledge	✓	✓	✓
Using resources: instruction: content experts			✓
Using resources: instruction: experienced teachers	✓		
Using resources: instruction: lesson plans(others)	✓		
Using resources: instruction: online	✓		
Using resources: instruction: textbooks			✓
Using resources: math content: instructor			✓
Using resources: math content: online			✓
Valuing "aha moment" in instruction		✓	
Valuing ability to anticipate difficulty: teaching	✓		
Valuing ability to explain clearly	✓		
Valuing ability to understand students		✓	
Valuing applications: teaching		✓	
Valuing conceptual understanding: teaching			✓
Valuing connections: teaching			✓

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Table G.1 – continued from previous page			
Code	**Didactical**	**Pedagogical**	**Subject-Matter**
Valuing content knowledge: instruction			✓
Valuing didactical skill in teaching	✓		
Valuing knowledge of students to inform instruction		✓	
Valuing making math accessible		✓	
Valuing multiple approaches: instruction	✓		
Valuing one-on-one instruction		✓	
Valuing patience	✓		
Valuing personalized instruction paths	✓		
Valuing preparation: teaching	✓		
Valuing recall			✓
Valuing skill practice	✓		
Viewing teaching as knowledge/skill transmission			✓
Wanting students to pay attention	✓		

Table G.1: Table of Codes Assigned to Categories within Beijaard et al.'s Framework.

Appendix H Code Categories Adapted from Van Zoest & Bohl's Model

Code	Content (SIM)	Pedagogy (SIM)	Participation (SIM)	Perception (SIC)	Competence (SIC)
Attempting to change student attitude		✓			
Connecting instruction to course level			✓		
Connecting to larger curriculum			✓		
Devaluing brute-force/guess-and-check	✓				
Devaluing existing GTA prep course					✓
Developing instructional strategies: working examples	✓				
Displaying gap in content understanding: teaching	✓				✓
Distancing self from students				✓	
Enjoying procedural manipulations	✓				
Equating "real classroom" with delivering same instruction to all students at same time		✓			
Equating big school with devaluing teaching				✓	
Equating confidence with excitement				✓	
Equating inability to explain with student inability to learn		✓		✓	
Equating lack of recall with dislike of topic: teaching	✓				

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Table H.1 – continued from previous page					
Code	**Content (SIM)**	**Pedagogy (SIM)**	**Participation (SIM)**	**Perception (SIC)**	**Competence (SIC)**
Equating teacher competence with student mastery/retention				✓	
Equating teaching with content delivery		✓			
Excluding supervising teachers as core figures: implicit				✓	
Expressing concern over struggling students		✓			
Expressing confidence in content knowledge: teaching	✓	✓			
Expressing enjoyment of teaching				✓	
Expressing fear of public speaking				✓	
Expressing frustration: teaching				✓	
Fearing being seen as incompetent				✓	
Fearing making mathematical error: teaching	✓			✓	
Feeling pressure/stress: teaching					✓
Focusing on individual learner needs		✓			
Handling student confusion: assume student fault		✓			
Handling student confusion: assume teacher fault		✓			
Handling student confusion: different approach/explanation		✓			
Handling student confusion: explain again		✓			
Handling student confusion: focus on student affect		✓			

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Table H.1 – continued from previous page					
Code	**Content (SIM)**	**Pedagogy (SIM)**	**Participation (SIM)**	**Perception (SIC)**	**Competence (SIC)**
Handling student confusion: remain positive		✓			
Handling student confusion: remind process/rule	✓				
Handling teaching discomfort: avoid/delay					✓
Handling teaching discomfort: doing best					✓
Handling teaching discomfort: relying on prepared notes	✓				
Handling teaching discomfort: thinking about big picture	✓				
Identifying gaps in preparation to teach		✓			
Identifying peers as core figures: teaching					✓
Identifying professors as core figures: teaching					✓
Identifying supervising teachers as core figures: teaching					✓
Lesson study role: content					✓
Lesson study role: equitable tasks					✓
Lesson study role: leadership					✓
Lesson study role: lesson delivery					✓
Lesson study role: student assessment					✓
Lesson study role: using knowledge of lab to inform lesson plan					✓
Lesson study role: using knowledge of students to inform lesson plan					✓
Perceiving competence: teaching				✓	
Continued on next page					

Table H.1 – continued from previous page					
Code	**Content (SIM)**	**Pedagogy (SIM)**	**Participation (SIM)**	**Perception (SIC)**	**Competence (SIC)**
Perceiving content gap in ALEKS	✓				
Perceiving disconnect between peer/faculty: teaching attitude				✓	
Perceiving growth in competence: didactical		✓		✓	
Perceiving growth in competence: pedagogical		✓		✓	
Perceiving growth in competence: unpacking math	✓			✓	
Perceiving little or no change as teacher				✓	
Perceiving others' perceptions: incompetence				✓	
Perceiving others' perceptions: weirdness				✓	
Questioning balance of why and how in teaching math	✓				
Questioning own competence: teaching				✓	
Receiving message from peers: balance					✓
Receiving message from professors: de-emphasize teaching				✓	
Reflecting on practice: assessing lesson		✓			
Reflecting on practice: assessing student understanding		✓			
Reflecting on practice: content	✓				
Reflecting on practice: conveying social knowledge	✓				
Reflecting on practice: didactical basics					✓
Reflecting on practice: instructional options					✓
Continued on next page					

Table H.1 – continued from previous page					
Code	**Content (SIM)**	**Pedagogy (SIM)**	**Participation (SIM)**	**Perception (SIC)**	**Competence (SIC)**
Reflecting on practice: pacing					✓
Reflecting on practice: procedure					✓
Reflecting on practice: questioning techniques		✓			✓
Reflecting on practice: scaffolding/prerequisite knowledge	✓	✓			
Reflecting on practice: student engagement					✓
Reflecting on practice: technology		✓			
Reflecting on practice: wait time				✓	
Revoicing student attitude: negative		✓			
Unpacking content knowledge	✓	✓			
Using resources: instruction: experienced teachers					✓
Using resources: instruction: lesson plans(others)					✓
Using resources: instruction: online					✓
Using resources: instruction: peers					✓
Using resources: instruction: textbooks					✓
Using resources: math content: online	✓				
Valuing “aha moment” in instruction		✓			
Valuing ability to anticipate difficulty: teaching				✓	
Valuing ability to explain clearly		✓		✓	
Valuing ability to help others: math		✓		✓	
Valuing ability to understand students		✓			
Continued on next page					

Table H.1 – continued from previous page					
Code	**Content (SIM)**	**Pedagogy (SIM)**	**Participation (SIM)**	**Perception (SIC)**	**Competence (SIC)**
Valuing applications: teaching		✓			
Valuing communication: teaching		✓			
Valuing competence: teaching				✓	
Valuing conceptual understanding: teaching	✓				
Valuing confidence: teaching				✓	
Valuing connections: teaching	✓				
Valuing content knowledge: instruction	✓				
Valuing didactical skill in teaching		✓			
Valuing experience: teaching					✓
Valuing interaction between teacher and students				✓	
Valuing knowledge of students to inform instruction		✓		✓	
Valuing low risk environment for early teaching experience			✓		✓
Valuing making math accessible		✓		✓	
Valuing multiple approaches: instruction		✓			
Valuing observing other teachers				✓	
Valuing one-on-one instruction		✓			
Valuing opinions of others				✓	
Valuing patience		✓			
Valuing personalized instruction paths			✓		
Valuing positive teacher attitude				✓	

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Table H.1 – continued from previous page					
Code	**Content (SIM)**	**Pedagogy (SIM)**	**Participation (SIM)**	**Perception (SIC)**	**Competence (SIC)**
Valuing preparation: teaching		✓			✓
Valuing recall	✓				✓
Valuing skill practice					✓
Viewing teaching as a process of improvement					✓
Viewing teaching as knowledge/skill transmission	✓				
Wanting students to enjoy math				✓	
Wanting students to pay attention				✓	

Table H.1: Table of codes assigned to categories adapted from Van Zoest & Bohl's framework for teacher identity development.

Appendix I Code Categories Adapted from Ronfeldt & Grossman's Model

Code	**Prof Images (Teaching)**	**Prof Images (Math)**	**Teaching Feedback**	**Coursework Feedback**	**Research Feedback**	**Provisional Selves**
Addressing research expectations					✓	
Devaluing existing GTA prep course			✓			
Disconnecting math skills from PCK	✓	✓				
Disliking research					✓	✓
Disliking theory				✓		✓
Enjoying problem solving						✓
Equating "real classroom" with delivering same instruction to all students at same time	✓					
Equating big school with devaluing teaching	✓	✓				
Equating math success with natural gift		✓				
Equating mathematician with generating knowledge		✓				
Equating mathematician with problem solving		✓				✓
Equating teacher competence with student mastery/retention	✓					✓
Equating teaching with content delivery	✓					

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Table I.1 – continued from previous page						
Code	**Prof Images (Teaching)**	**Prof Images (Math)**	**Teaching Feedback**	**Coursework Feedback**	**Research Feedback**	**Provisional Selves**
Excluding supervising teachers as core figures: implicit			✓			
Expressing “calling” to be teacher						✓
Expressing “calling” to get Masters						✓
Expressing confidence in content knowledge: teaching			✓			
Expressing enjoyment/love of math						✓
Expressing enjoyment of teaching	✓					✓
Expressing fear of public speaking			✓			
Expressing frustration: teaching			✓			✓
Fearing being seen as incompetent			✓			✓
Fearing making mathematical error: teaching			✓			
Feeling frustration: math coursework				✓		✓
Feeling overwhelmed: workload				✓		✓
Feeling pressure/stress: teaching			✓			✓
Focusing on grades: own				✓		
Identifying contact out of class as part of teaching	✓					

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Table I.1 – continued from previous page						
Code	**Prof Images (Teaching)**	**Prof Images (Math)**	**Teaching Feedback**	**Coursework Feedback**	**Research Feedback**	**Provisional Selves**
Identifying grading as part of teaching	✓					
Identifying lesson planning as part of teaching	✓					
Identifying professors as core figures: teaching			✓			
Identifying supervising teachers as core figures: teaching			✓			
Perceiving competence: teaching			✓			
Perceiving disconnect between peer/faculty: teaching attitude			✓			
Perceiving growth in competence: didactical			✓			
Perceiving growth in competence: pedagogical			✓			
Perceiving growth in competence: unpacking math			✓			
Perceiving little or no change as teacher			✓			
Perceiving others' perceptions: incompetence			✓			
Perceiving others' perceptions: weirdness			✓			
Projecting ahead: Future math: ambivalence		✓				✓
Projecting ahead: Future math: negative		✓				✓
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Table I.1 – continued from previous page						
Code	**Prof Images (Teaching)**	**Prof Images (Math)**	**Teaching Feedback**	**Coursework Feedback**	**Research Feedback**	**Provisional Selves**
Projecting ahead: Future math: positive		✓				✓
Projecting ahead: Future teaching: ambivalence						✓
Projecting ahead: Future teaching: negative						✓
Projecting ahead: Future teaching: positive	✓					✓
Projecting ahead: Past math: ambivalence						✓
Projecting ahead: Past math: negative		✓				✓
Projecting ahead: Past math: positive	✓	✓				✓
Projecting ahead: Past teaching: negative						✓
Projecting ahead: Past teaching: positive	✓					✓
Projecting ahead: Planning academic career	✓	✓				✓
Projecting ahead: Planning nonacademic career		✓				✓
Questioning nature of mathematics		✓				✓
Questioning own competence: mathematical		✓		✓		✓
Questioning own competence: teaching			✓			✓
Receiving message from peers: balance			✓	✓	✓	

Continued on next page

Table I.1 – continued from previous page						
Code	**Prof Images (Teaching)**	**Prof Images (Math)**	**Teaching Feedback**	**Coursework Feedback**	**Research Feedback**	**Provisional Selves**
Receiving message from professors: communication				✓	✓	
Receiving message from professors: de-emphasize teaching			✓		✓	
Receiving message from professors: know content			✓			
Receiving message from professors: value teaching		✓	✓			
Reflecting on practice: wait time			✓			
Valuing "aha moment" in instruction	✓		✓			✓
Valuing ability to anticipate difficulty: teaching			✓			✓
Valuing ability to explain clearly			✓			✓
Valuing ability to help others: math						✓
Valuing ability to understand students	✓					
Valuing applications: math		✓				✓
Valuing applications: teaching	✓					
Valuing asking for help				✓	✓	
Valuing balance/flexibility						✓
Valuing breadth in mathematics		✓				✓
Valuing challenge		✓				✓
Valuing communication: math		✓				

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Table I.1 – continued from previous page						
Code	**Prof Images (Teaching)**	**Prof Images (Math)**	**Teaching Feedback**	**Coursework Feedback**	**Research Feedback**	**Provisional Selves**
Valuing communication: teaching	✓					
Valuing competence: math		✓				
Valuing competence: teaching			✓			
Valuing conceptual understanding: teaching			✓			
Valuing confidence: teaching			✓			
Valuing connections: math				✓	✓	✓
Valuing connections: teaching			✓			✓
Valuing consistency in grading			✓			
Valuing content knowledge: instruction	✓		✓			
Valuing creation of knowledge		✓				✓
Valuing didactical skill in teaching			✓			
Valuing experience: teaching			✓			✓
Valuing interaction between teacher and students	✓		✓			✓
Valuing knowledge of students to inform instruction			✓			
Valuing logical thought		✓				
Valuing low risk environment for early teaching experience			✓			✓
Valuing making math accessible			✓			✓
Continued on next page						

Table I.1 – continued from previous page						
Code	**Prof Images (Teaching)**	**Prof Images (Math)**	**Teaching Feedback**	**Coursework Feedback**	**Research Feedback**	**Provisional Selves**
Valuing multiple approaches: instruction			✓			✓
Valuing multiple approaches: math				✓	✓	
Valuing observing other teachers			✓			✓
Valuing open ended problems		✓			✓	
Valuing opinions of others						✓
Valuing patience			✓			✓
Valuing peers: math						✓
Valuing personalized instruction paths	✓					✓
Valuing positive teacher attitude	✓					✓
Valuing preparation: teaching			✓			✓
Valuing problem-solving		✓				✓
Valuing recall		✓				
Valuing respect/obedience						✓
Valuing spiritual/religious						✓
Valuing teamwork				✓		✓
Viewing mathematics as difficult				✓		✓
Viewing mathematics as lots of work				✓		✓
Viewing mathematics as more than teaching					✓	✓
Viewing mathematics as superior to other fields		✓				
Viewing teaching as a process of improvement			✓			✓

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Table I.1 – continued from previous page						
Code	**Prof Images (Teaching)**	**Prof Images (Math)**	**Teaching Feedback**	**Coursework Feedback**	**Research Feedback**	**Provisional Selves**
Viewing teaching as knowledge/skill transmission			✓			✓
Viewing teaching as part of being academic mathematician	✓	✓				✓
Viewing teaching as preferable to grading	✓	✓				✓
Wanting to emulate a role model: teaching	✓					
Wanting to teach advanced students						✓

Table I.1: Table of codes assigned to categories adapted from Ronfeldt & Grossman's framework for professional identity development.

Appendix J Complete Tables of Differing Survey Responses

Differing Pre- and Post-Survey Responses for Anna				
	Flip?	**Prompt**	**Pre-**	**Post-**
Images (Math)	✓	I remember most of the things I learn in mathematics.	Agree	Disagree
		My math professors show little interest in the students.	Strongly Disagree	Disagree
		My mathematics teachers know when we are having trouble with our work.	Strongly Agree	Agree
		Working mathematics problems is fun.	Strongly Agree	Agree
		I feel at ease in a mathematics class.	Strongly Agree	Agree
		I often think, "I can't do it," when a mathematics problem seems hard.	Strongly Disagree	Disagree
		I enjoy talking to other people about mathematics.	Strongly Agree	Agree
		I am good at working mathematics problems.	Strongly Agree	Agree
		You can get along perfectly well in everyday life without mathematics.	Strongly Disagree	Disagree
		I would rather be given the right answer to a math problem than work it out myself.	Strongly Disagree	Disagree
		Most of the ideas in mathematics aren't very useful.	Strongly Disagree	Disagree
		I will be happy when I am done taking math classes.	Strongly Disagree	Disagree
		I think of myself as a mathematician.	Agree	Strongly Agree
Images (Teaching)	✓	Math professors spend very little time thinking about teaching.	Agree	Disagree
	✓	Math professors are expected to spend most of their time on research.	Disagree	Agree
	✓	I feel at ease talking about teaching.	Disagree	Agree
	✓	I get bored when people talk about different ways to teach.	Agree	Disagree
	✓	No matter how hard I try, there are some math topics I cannot teach well.	Disagree	Agree
	✓	I enjoy talking to other people about teaching.	Disagree	Agree
	✓	I am good at teaching.	Disagree	Agree
	✓	It scares me to have to teach math.	Disagree	Agree
	✓	I would be happy if I never taught math.	Disagree	Agree
	✓	I think of myself as a teacher.	Disagree	Agree
		College math professors are expected to teach well.	Strongly Agree	Agree

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Table J.1 – continued from previous page				
	Flip?	**Prompt**	**Pre-**	**Post-**
		It would make me sad if I never got to teach math.	Strongly Agree	Agree
		I don't like anything about teaching.	Disagree	Strongly Disagree
		I have a real desire to teach.	Strongly Agree	Agree
		If a student doesn't understand what I taught them, s/he must not have paid attention.	Disagree	Strongly Disagree
Beliefs (Teaching)	✓	Students should be allowed to invent ways to solve math problems before the teacher demonstrates how to solve the problems.	Agree	Disagree
	✓	The instructional scope and sequence of math topics should be determined by the formal organization of mathematics.	Disagree	Agree
	✓	Students should master computational procedures before they are expected to understand how those procedures work.	Disagree	Agree
	✓	Students learn mathematics best by figuring out for themselves the ways to find answers to math problems.	Disagree	Agree
	✓	Students should be told to solve problems the way the teacher has taught them.	Agree	Disagree
		Students should understand computational procedures before they master them.	Strongly Agree	Agree
		When selecting the next topic to be taught, a significant consideration is what students already know.	Agree	Strongly Agree
		Children will not understand multiplication and division until they have mastered some basic math facts.	Strongly Disagree	Disagree

Table J.1: Pre- and post-survey prompts for which Anna's responses differed. The survey was forced-choice with four options: Strongly Agree, Agree, Disagree, Strongly Disagree. All items with differing responses are included. Items that changed from Agree to Disagree or vice versa are indicated by a checkmark (✓) in the 'Flip?' column. Items that changed from (Strongly) Agree to (Strongly) Disagree or vice versa are indicated by a double checkmark (✓✓) in the 'Flip?' column.

Differing Pre- and Post-Survey Responses for Bill				
	Flip?	**Prompt**	**Pre-**	**Post-**
Images (Math)	✓✓	I would like to spend less time in school doing mathematics.	Strongly Disagree	Agree
	✓	I will be happy when I am done taking math classes.	Disagree	Agree
		Mathematics is useful for the problems of everyday life.	Agree	Strongly Agree
		My math professors show little interest in the students.	Strongly Disagree	Disagree
		Mathematics is helpful in understanding today's world.	Agree	Strongly Agree
		My mathematics teachers don't like students to ask questions.	Strongly Disagree	Disagree
		I feel tense when someone talks to me about mathematics.	Disagree	Strongly Disagree
		I often think, "I can't do it," when a mathematics problem seems hard.	Disagree	Strongly Disagree
		It is important to know mathematics in order to get a good job.	Agree	Strongly Agree
		It doesn't disturb me to work mathematics problems.	Agree	Strongly Agree
		I am good at working mathematics problems.	Strongly Agree	Agree
		I would rather be given the right answer to a math problem than work it out myself.	Strongly Disagree	Disagree
		Most of the ideas in mathematics aren't very useful.	Disagree	Strongly Disagree
Images (Teaching)	✓✓	The only reason I am teaching is because I have to.	Strongly Disagree	Agree
	✓	I would be happy if I never taught math.	Disagree	Agree
	✓	I have a real desire to teach.	Agree	Disagree
	✓	I enjoy talking to other people about teaching.	Agree	Disagree
	✓	Math professors are expected to spend most of their time on research.	Agree	Strongly Disagree
		I feel tense when someone talks to me about teaching math.	Strongly Disagree	Disagree
		I don't like anything about teaching.	Disagree	Strongly Disagree
		No matter how hard I try, there are some math topics I cannot teach well.	Strongly Agree	Agree
		It doesn't disturb me to teach a math class.	Strongly Agree	Agree
		I am good at teaching.	Strongly Agree	Agree
		I get a feeling of satisfaction from teaching.	Agree	Strongly Agree

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Table J.2 – continued from previous page				
	Flip?	**Prompt**	**Pre-**	**Post-**
		I remember most of the things that happen when I teach.	Agree	Strongly Agree
		I would rather watch someone else teach than teach a topic myself.	Agree	Strongly Agree
		If a student doesn't understand what I taught them, s/he must not have paid attention.	Strongly Agree	Disagree
Beliefs (Teaching)	✓	Time should be spent practicing computational procedures before students are expected to understand the procedures.	Agree	Disagree
	✓	Children should understand the meaning of multiplication and division before they memorize basic math facts.	Disagree	Agree
	✓	Students learn mathematics best by figuring out for themselves the ways to find answers to math problems.	Disagree	Agree
	✓	Children will not understand multiplication and division until they have mastered some basic math facts.	Agree	Disagree
		Students should understand computational procedures before they master them.	Agree	Strongly Agree
		The natural development of students' mathematical ideas must be considered in making instructional decisions.	Agree	Strongly Agree
		To be successful in mathematics, a student must be a good listener.	Strongly Agree	Agree
		Mathematics should be presented to students in such a way that they can discover relationships for themselves.	Agree	Strongly Agree
		Students should be told to solve problems the way the teacher has taught them.	Strongly Disagree	Disagree

Table J.2: Pre- and post-survey prompts for which Bill's responses differed. The survey was forced-choice with four options: Strongly Agree, Agree, Disagree, Strongly Disagree. All items with differing responses are included. Items that changed from Agree to Disagree or vice versa are indicated by a checkmark (✓) in the 'Flip?' column. Items that changed from (Strongly) Agree to (Strongly) Disagree or vice versa are indicated by a double checkmark (✓✓) in the 'Flip?' column.

Differing Pre- and Post-Survey Responses for Cora				
	Flip?	**Prompt**	**Pre-**	**Post-**
Images (Math)	✓	I like to do outside reading in mathematics.	Disagree	Agree
	✓	Sometimes I read ahead in my mathematics texts.	Agree	Disagree
		I usually understand what we are talking about in math class.	Strongly Agree	Agree
		My mathematics teachers don't like students to ask questions.	Disagree	Strongly Disagree
		I often think, "I can't do it," when a mathematics problem seems hard.	Strongly Disagree	Disagree
		It is important to know mathematics in order to get a good job.	Strongly Agree	Agree
		I would rather be given the right answer to a math problem than work it out myself.	Strongly Disagree	Disagree
		Most of the ideas in mathematics aren't very useful.	Strongly Disagree	Disagree
		My mathematics professors don't seem to enjoy teaching.	Strongly Disagree	Disagree
Images (Teaching)	✓	Math professors spend very little time thinking about teaching.	Disagree	Agree
	✓	No matter how hard I try, there are some math topics I cannot teach well.	Disagree	Agree
	✓	Math professors are expected to spend most of their time on research.	Disagree	Agree
		I have a real desire to teach.	Agree	Strongly Agree
		I think of myself as a teacher	Strongly Agree	Agree
		It would make me sad if I never got to teach math.	Strongly Agree	Agree
Beliefs (Teaching)	✓✓	Students should understand computational procedures before they master them.	Strongly Agree	Disagree
	✓	Students should be allowed to invent ways to solve math problems before the teacher demonstrates how to solve the problems.	Disagree	Agree
	✓	Students can figure out ways to solve many math problems without formal instruction.	Disagree	Agree
	✓	Students should solve mathematical problems before they master computational procedures.	Agree	Disagree
	✓	The teacher should demonstrate how to solve math problems before students are allowed to solve problems.	Agree	Disagree
	✓	Children will not understand multiplication and division until they have mastered some basic math facts.	Agree	Disagree
		The instructional sequence of math topics should be determined by the order in which students naturally acquire math concepts.	Strongly Agree	Agree

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Table J.3 – continued from previous page				
	Flip?	**Prompt**	**Pre-**	**Post-**
		Students learn mathematics best from the teacher's demonstrations and explanations.	Strongly Agree	Agree
		To be successful in mathematics, a student must be a good listener.	Strongly Agree	Agree
		Students learn math best by attending to the teacher's explanation of how to do the activity.	Strongly Agree	Agree
		When selecting the next topic to be taught, one must consider the logical organization of mathematics.	Strongly Agree	Agree
		Students should be told to solve problems the way the teacher has taught them.	Strongly Agree	Agree

Table J.3: Pre- and post-survey prompts for which Cora's responses differed. The survey was forced-choice with four options: Strongly Agree, Agree, Disagree, Strongly Disagree. All items with differing responses are included. Items that changed from Agree to Disagree or vice versa are indicated by a checkmark (✓) in the 'Flip?' column. Items that changed from (Strongly) Agree to (Strongly) Disagree or vice versa are indicated by a double checkmark (✓✓) in the 'Flip?' column.

Differing Pre- and Post-Survey Responses for Dave				
	Flip?	**Prompt**	**Pre-**	**Post-**
Images (Math)	✓✓	Most of the ideas in mathematics aren't very useful.	Agree	Strongly Disagree
	✓	My mathematics teachers present material in a clear way.	Agree	Disagree
	✓	I often think, "I can't do it," when a mathematics problem seems hard.	Disagree	Agree
	✓	My mathematics professors don't seem to enjoy teaching.	Disagree	Agree
		Mathematics is useful for the problems of everyday life.	Strongly Agree	Agree
		Mathematics is something I enjoy very much.	Strongly Agree	Agree
		I don't do very well in my mathematics courses.	Disagree	Strongly Disagree
		When I hear the word "mathematics" I have a feeling of dislike.	Strongly Disagree	Disagree
		Sometimes I read ahead in my mathematics texts.	Strongly Disagree	Disagree
		Mathematics is helpful in understanding today's world.	Strongly Agree	Agree
		No matter how hard I try, I cannot understand mathematics.	Strongly Disagree	Disagree
		It is important to know mathematics in order to get a good job.	Agree	Strongly Agree

Continued on next page

Table J.4 – continued from previous page				
	Flip?	**Prompt**	**Pre-**	**Post-**
		It doesn't disturb me to work mathematics problems.	Agree	Strongly Agree
		It makes me nervous to think about doing mathematics.	Strongly Disagree	Disagree
		I will be happy when I am done taking math classes.	Disagree	Strongly Disagree
Images (Teaching)	✓✓	No matter how hard I try, there are some math topics I cannot teach well.	Strongly Agree	Disagree
	✓✓	Math professors are expected to spend most of their time on research.	Strongly Disagree	Agree
	✓	Math professors spend very little time thinking about teaching.	Disagree	Agree
		I feel tense when someone talks to me about teaching math.	Strongly Disagree	Disagree
		I look forward to teaching math.	Strongly Agree	Agree
		I feel at ease talking about teaching.	Strongly Agree	Agree
		It doesn't disturb me to teach a math class.	Strongly Agree	Agree
		I enjoy talking to other people about teaching.	Strongly Agree	Agree
		I have a good feeling towards teaching.	Strongly Agree	Agree
		I get a feeling of satisfaction from teaching.	Strongly Agree	Agree
		It is important to me to teach well.	Strongly Agree	Agree
		I would rather watch someone else teach than teach a topic myself.	Strongly Disagree	Disagree
		I often think, "I can't do it,' when a student doesn't understand what I am teaching.	Strongly Disagree	Disagree
		I have a real desire to teach.	Strongly Agree	Agree
		I think of myself as a teacher	Strongly Agree	Agree
		If a student doesn't understand what I taught them, s/he must not have paid attention.	Strongly	Disagree
Beliefs (Teaching)	✓	Students should solve mathematical problems before they master computational procedures.	Disagree	Agree
	✓	Children should understand the meaning of multiplication and division before they memorize basic math facts.	Disagree	Agree
	✓	Students should understand computational procedures before they master them.	Disagree	Agree

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Table J.4 – continued from previous page				
	Flip?	**Prompt**	**Pre-**	**Post-**
	✓	Children will not understand multiplication and division until they have mastered some basic math facts.	Agree	Disagree
	✓	Time should be spent practicing computational procedures before students are expected to understand the procedures.	Agree	Disagree
		The instructional sequence of math topics should be determined by the order in which students naturally acquire math concepts.	Agree	Strongly Agree
		The natural development of students' mathematical ideas must be considered in making instructional decisions.	Agree	Strongly Agree
		The instructional scope and sequence of math topics should be determined by the formal organization of mathematics.	Strongly Agree	Agree
		Students learn math best by attending to the teacher's explanation of how to do the activity.	Strongly Agree	Agree
		When selecting the next topic to be taught, one must consider the logical organization of mathematics.	Strongly Agree	Agree

Table J.4: Pre- and post-survey prompts for which Dave's responses differed. The survey was forced-choice with four options: Strongly Agree, Agree, Disagree, Strongly Disagree. All items with differing responses are included. Items that changed from Agree to Disagree or vice versa are indicated by a checkmark (✓) in the 'Flip?' column. Items that changed from (Strongly) Agree to (Strongly) Disagree or vice versa are indicated by a double checkmark (✓✓) in the 'Flip?' column.

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